September P2 2015 **MEMORANDUM** **Grade/Graad 12**

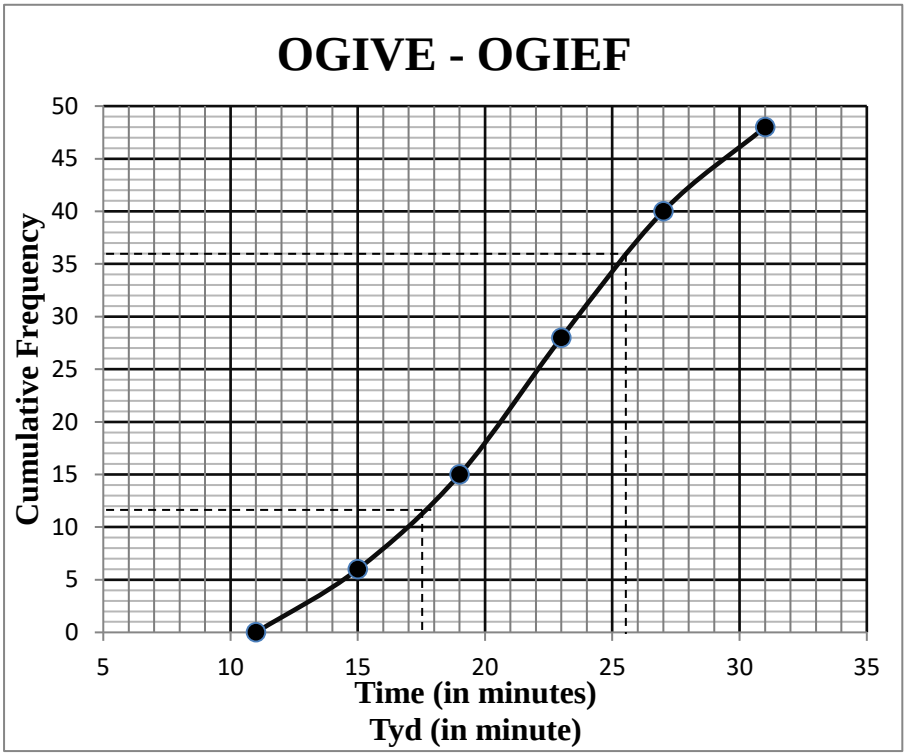
NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.*
- *Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.*

VRAAG/QUESTION 1

1.1	$a = 16{,}16$ (16,1571639) $b = 0{,}88$ (0,8847043972) answ only:full marks $\hat{y} = 16{,}16 + 0{,}88x$	✓ a ✓ b ✓ equation	(3)
1.2	$\hat{y} = 16{,}157... + 0{,}884...(73)$ answ only:full marks $= 81\%$	✓ Subst into eq ✓ Answer (accept 80%)	(2)
1.3	$r = 0.92$ (0.9177373213)	✓✓ Answer (accept 0,91)	(2)
1.4	Yes./Ja The correlation between the two exams is very strong/ Die korrelasie tussen die twee eksamens is baie sterk..	✓ Yes ✓ correct justification	(2)
			[9]

QUESTION/ VRAAG 2

2.1	Modal Class: $19 \leq t < 23$	✓ ✓ answer	(1)
2.2		✓ grounding at (11;0) ✓ plotting upper limits ✓ plotting cumulative frequency ✓ drawing a smooth curve ✓ geanker by (11;0) ✓ plot boonste limiete ✓ plot kumulatiewe frekwensie ✓ gladde kurwe	(4)
2.3	Q_1 = 17,5 (accept 17 – 18) Q_3 = 25,5 (accept 25 – 26) IQR/IKO = 25,5 – 17,5 = 8 (accept 7 – 9)	✓17,5 ✓25,5 ✓answer	(3)
2.4	Read off from graph: 18 occasions ∴ 48 – 18 = 30 times answ only:full marks	✓ accept 17 – 18 ✓ answer	(2)
			[10]

QUESTION/ VRAAG 3

3.1	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BC} = \frac{0+8}{-5-1}$ $m_{BC} = \frac{8}{-6} = -\frac{4}{3}$	✓ Substitution ✓ Answer	(2)
3.2	$E = (\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2})$ $= \left(\frac{-5+1}{2} ; \frac{0-8}{2}\right)$ $= (-2; -4)$	✓ *x* value ✓ *y* value	(2)
3.3	$m_{DE} = \frac{3}{4}$ [DE ⊥BC] $y-(-4) = \frac{3}{4}(x-(-2))$ *OR* $-4 = \frac{3}{4}(-2)+c$ $y = \frac{3}{4}x - 2\frac{1}{2}$	✓ m_{DE} ✓ Substitute m_{DE} and E(–2 ; –4) ✓ equation	(3)
3.4	$\tan\theta = m_{AD}$ $\tan\theta = -\frac{4}{3}$ $\theta = 180° - 53{,}13°$ $= 126{,}87°$	✓ $\tan\theta = -\frac{4}{3}$ ✓ 53,13° ✓ answer (obtuse)	(3)
3.5	$O\hat{F}D = 126{,}87° - 90°$ [ext ∠ of Δ] $= 36{,}87°$	✓ method ✓ answer	(2)
3.6	$(5\sqrt{2})^2 = (-5-x)^2 + (0-7)^2$ $50 = (-5-x)^2 + (0-7)^2$ $50 = x^2 + 10x + 25 + 49$ $x^2 + 10x + 24 = 0$ $(x+6)(x+4) = 0$ $x \neq -6$ or $x = -4$	✓ Subst into dist formula ✓ $AB^2 = 50$ ✓ standard form ✓ factors ✓ correct choice	(5)
3.7	$(x-a)^2 + (y-b)^2 = r^2$ $(x-(-2))^2 + (y-(-4))^2 = (-5-(-2))^2 + (0-(-4))^2$ $(x+2)^2 + (y+4)^2 = (-5+2)^2 + (0+4)^2$ $(x+2)^2 + (y+4)^2 = 25$ OR $(x+2)^2 + (y+4)^2 = r^2$ $(-5+2)^2 + (0+4)^2 = r^2$ $(x+2)^2 + (y+4)^2 = 25$ OR $(x+2)^2 + (y+4)^2 = r^2$ $(1+2)^2 + (-8+4)^2 = r^2$	✓ Subst LHS ✓ Subst RHS ✓ Answer ✓ Subst LHS ✓ Subst (–5 ; 0) ✓ Answer ✓ Subst LHS ✓ Subst (1 ; –8)	(3)

	$(x+2)^2+(y+4)^2=25$	✓ Answer	
			[20]

QUESTION/ VRAAG 4

4.1.1	$x^2+y^2+4x-4y-12=0$ $(x+2)^2+(y-2)^2=12+4+4=20$ M(–2 ; 2) OR $M(\frac{4}{-2};\frac{-4}{-2})$ M(–2 ; 2) answ only:full marks	✓ *x* value ✓ *y* value ✓ *x* value ✓ *y* value	(2)
4.1.2	$x^2+0^2+4x-4(0)-12=0$ $x^2+4x-12=0$ $(x+6)(x-2)=0$ $x\neq -6$ and $x=2$, C(2 ; 0) OR $\frac{x_C-6}{2}=-2$ $x_C=2$ C(2 ; 0) answ only:full marks	✓ *y*- coordinate = 0 ✓ factors ✓ correct *x* value ✓ correct use of formula ✓ *x* value ✓ *y* = 0	(3)
4.2	$m_{MC}=-\frac{1}{2}$ $m_{tangent}=2$ [tangent ⊥ diameter] $y-y_1=m(x-x_1)$ $y-4=2(x-(-6))$ $y=2x+16$	✓ S ✓R ✓ substituting A(–6 ; 4) ✓ equation	(4)
4.3	B(*x* ; 0) $y=2x+16$ $\therefore\ 0=2x+16$ $\therefore\ x=-8$ Area $\Delta ABC=\frac{1}{2}BC\times h$ $=\frac{1}{2}(10)\times 4$ = 20 sq units OR B(*x* ; 0) $y=2x+16$ $\therefore\ 0=2x+16$ $\therefore\ x=-8$ $AB=\sqrt{(-6+8)^2+(4-0)^2}$ $AB=\sqrt{20}$	✓ *y* = 0 in equation ✓ x_B ✓ BC = 10 ✓ *h* = 4 ✓ answer ✓ *y* = 0 in equation ✓ x_B	(5)

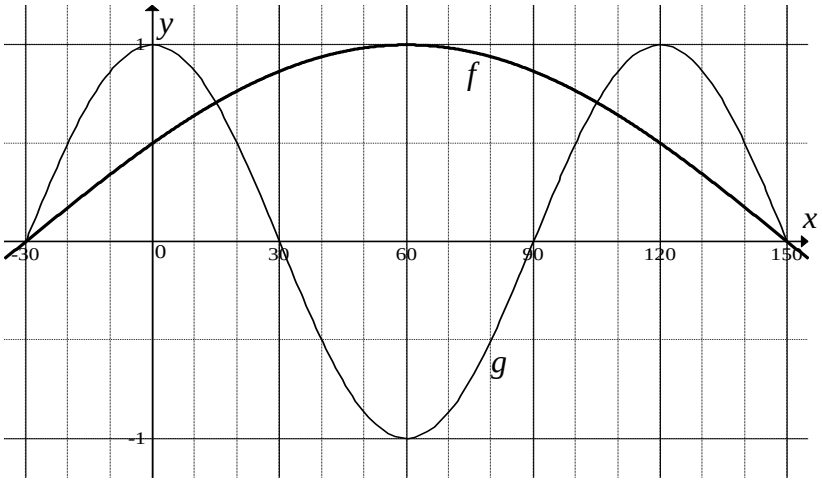
	$Area\ of\ \Delta = \frac{1}{2}(\text{AB})(\text{AC})$ $= \frac{1}{2}(\sqrt{20})(2\sqrt{20})$ $= 20\ unit^2$	✓ AB $= \sqrt{20}$ ✓ AC $= 2radius = 2\sqrt{20}$ ✓ answer	

4.4	Eq of tangent parallel to AB through C: $y - y_1 = m(x - x_1)$ $y - 0 = 2(x - 2)$ $y = 2x - 4$ $\therefore\ -4 < k < 16$ OR $x^2 + (2x + k)^2 + 4x - 4(2x + k) - 12 = 0$ $5x^2 + 4xk - 4x + k^2 - 4k - 12 = 0$ $5x^2 + (4k - 4)x + (k^2 - 4k - 12) = 0$ $\therefore\ \Delta > 0$ $(4k - 4)^2 - 4(5)(k^2 - 4k - 12) > 0$ $-4k^2 + 64k + 256 > 0$ $k^2 - 16k - 64 > 0$ $(k - 16)(k + 4) > 0$ $\therefore\ -4 < k < 16$	✓ subst C(2 ; 0) into equation ✓ eq of tangent through C ✓ –4 ✓ 16 ✓ between ✓ standard form ✓ $\Delta > 0$ ✓ –4 ✓ 16 ✓ between	(5)
			[19]

QUESTION/ VRAAG 5

5.1.1	$\sin 149^\circ = \sin 31^\circ$ $= p$	✓ reduction ✓ answer	(2)
5.1.2	$\cos(-59^\circ) = \cos 59^\circ$ $= \sin 31^\circ$ $= p$	✓ co-ratio ✓ answer	(2)
5.1.3	$\cos 62^\circ = 1 - 2\sin^2 31^\circ$ $= 1 - 2p^2$	✓ double formula ✓ answer in terms of p	(2)
5.2	$(-\tan\theta)(\cos\theta)^2 + (-\cos\theta)(\sin\theta)$ $\left(-\frac{\sin\theta}{\cos\theta}\right)(\cos\theta)^2 - (\cos\theta)(\sin\theta)$ $-(\cos\theta)(\sin\theta) - (\cos\theta)(\sin\theta)$ $-2(\cos\theta)(\sin\theta)$ $-\sin 2\theta$	✓ $-\tan\theta$ ✓ $\cos\theta$ ✓ $-\cos\theta$ ✓ identity ✓ simplify and add ✓ answer	(6)
5.3.1	$LHS = \frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1}$ $= \frac{2\sin x.\cos x + \sin x}{2\cos^2 x - 1 + \cos x + 1}$ $= \frac{\sin x\,(2\cos x + 1)}{2\cos^2 x + \cos x}$ $= \frac{\sin x\,(2\cos x + 1)}{\cos x\,(2\cos x + 1)}$ $= \frac{\sin x}{\cos x} = \tan x$ $= RHS$	✓ expansion of sin 2x ✓ correct expansion of cos 2x ✓ simplify numerator ✓ factorize numerator & denominator ✓ Identity	(5)
5.3.2	$\cos x\,(2\cos x + 1) = 0$ or $\tan x = \infty$ x = 240° or 270°	✓ 240° ✓ 270°	(2)
			[19]

QUESTION/ VRAAG 6

6.1	$\sin(x+30°) = \cos 3x$ $= \sin(90° - 3x)$ $x + 30° = 90° - 3x + k.360°$ $4x = 60° + k.360°$ $x = 15° + k.90°$ or $x + 30° = 180° - (90° - 3x) + k.360°$ $-2x = 60° + k.360°$ $x = -30° - k.180°\ ;\ k \in Z$ OR $\sin(x+30°) = \cos 3x$ $\cos[90° - (x+30)] = \cos 3x$ $90° - (x+30°) = 3x + k.360°$ $-4x = -60° + k.360°$ $x = 15° - k.90°$ or $90° - (x+30°) = 360° - 3x + k.360°$ $2x = 300° + k.360°$ $x = 150° + k.180°\ ;\ k \in Z$	✓ co-ratio ✓ equation ✓ general solution ✓ equation ✓ general solution ✓ k ∈ Z ✓ co-ratio ✓ equation ✓ general solution ✓ equation ✓ general solution ✓ k ∈ Z	(6)
6.2.1		✓ max tp's ✓ min tp ✓ x intercepts	(3)
6.2.2	Period = 120°	✓ answer	(1)
6.2.3	$x \in [15° ; 105°]$ OR $15° \le x \le 105°$	✓ 15° ✓ 105° ✓ notation	(3)
			[13]

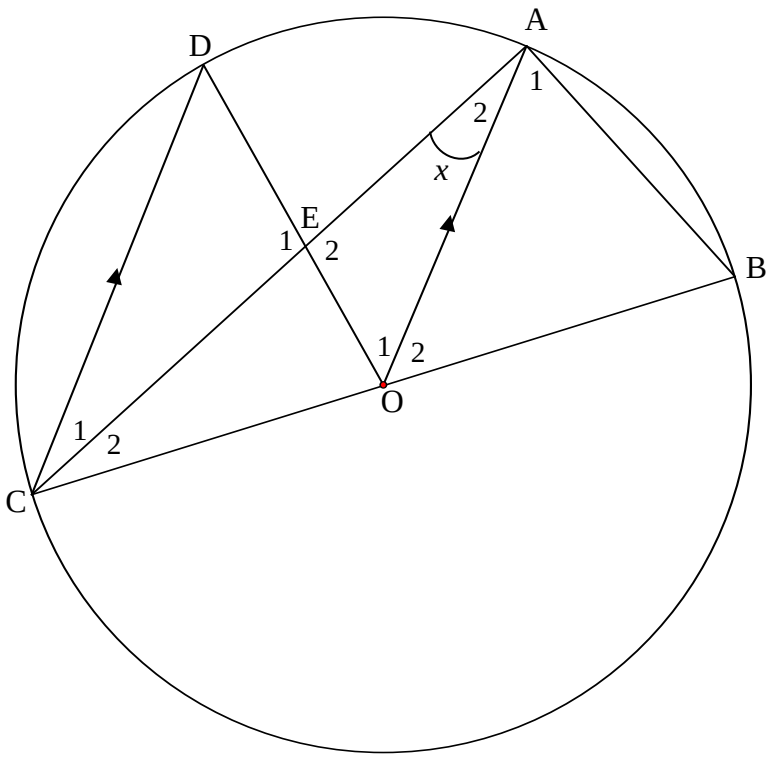
QUESTION 7

7.1	$\tan\alpha = \frac{AP}{AB}$ $AP = AB \tan\alpha$	✓ correct ratio ✓ AP ito AB and α	(2)
7.2	$\frac{AB}{\sin\beta} = \frac{20}{\sin(180° - (\theta + \beta))}$ $\frac{AB}{\sin\beta} = \frac{20}{\sin(\theta + \beta)}$ $\therefore AB = \frac{20\sin\beta}{\sin(\theta+\beta)}$ $\therefore AP = \frac{20.\sin\beta.\tan\alpha}{\sin(\theta + \beta)}$	✓ correct subst into sine rule ✓ reduction ✓ AB as subject and subst into7.1	(3)
7.3	$\theta = \beta$ [∠s opp equal sides] $AP = \frac{20.\sin\beta.\tan\alpha}{\sin 2\beta}$ $AP = \frac{20.\sin\beta.\tan\alpha}{2\sin\beta.\cos\beta}$ $AP = \frac{10.\tan\alpha}{\cos\beta}$	✓ replace θ ✓ expansion of sin 2β ✓ simplified answer	(3)
			[8]

QUESTION/ VRAAG 8

8.1	Line from centre to midpt of chord OR Line from centre bisects chord	✓ reason	(1)
8.2	$\hat{B} = 43°$ [∠s in same segment]	✓S ✓R	(2)
8.3	$B\hat{A}D = 90°$ [∠ in semi-circle] $= \hat{E}_1$	✓S ✓ R	(2)
8.4	BD = 28 units $\therefore \cos 43° = \frac{AB}{28}$ $\therefore$ AB = 28 cos 43° = 20,48 units	✓ correct ratio ✓ answer	(2)
			[7]

QUESTION/ VRAAG 9

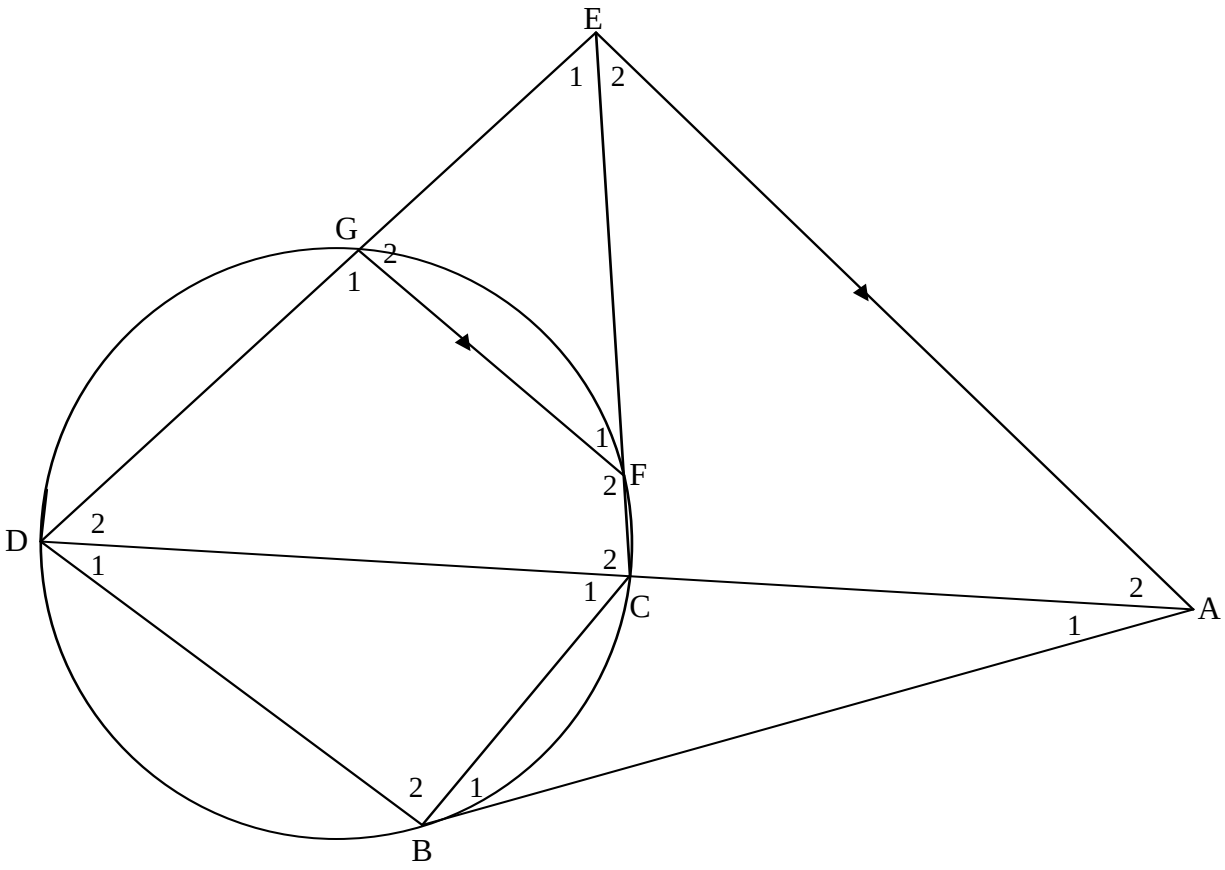


9.1.1	verwiss ∠e/alternate ∠s; CD | | OA	✓ R	(1)
9.1.2	CO = OA [radii] ∠s opp equal sides/∠e to gelyke sye	 ✓ R	(1)
9.2.1	$C\hat{A}B = 90°$ [∠ in semi-circle/halfsirkel] $\therefore\ \hat{A}_1 = 90° - x$	✓ S ✓R ✓ S	(3)
9.2.2	$\hat{O}_1 = 2x$ [∠ at centre = 2× ∠ at circum/midpts∠ = 2× omtr∠]	✓ S ✓R	(2)
9.2.3	$\hat{O}_2 = 2x$ [ext ∠ of ΔACO] OR $\hat{O}_2 = 2x$ [∠ at centre = 2× ∠ at circum/midpts∠ = 2× omtr∠] OR $\hat{A}_1 = 90° - x = \hat{B}$ [∠s opp equal sides/∠e to gelyke sye] $\therefore\ \hat{O}_2 = 180° - 2(90° - x) = 2x$ [∠s of Δ]	✓ S ✓R ✓ S ✓R ✓ R ✓ S	(2)
9.3	$E\hat{O}B = 4x$ [$\hat{O}_1 = 2x$ and $\hat{O}_2 = 2x$] $E\hat{O}B + E\hat{A}B = 180°$ [opp ∠s of cyc quad supp/ to ∠e v kdvh suppl] $\therefore\ 4x + 90° = 180°$ $x = 22,5°$	 ✓R ✓ equation ✓ answer	(3)
			[12]

QUESTION/ VRAAG 10

10.1	proportional/eweredig	✓ S	(1)

10.2

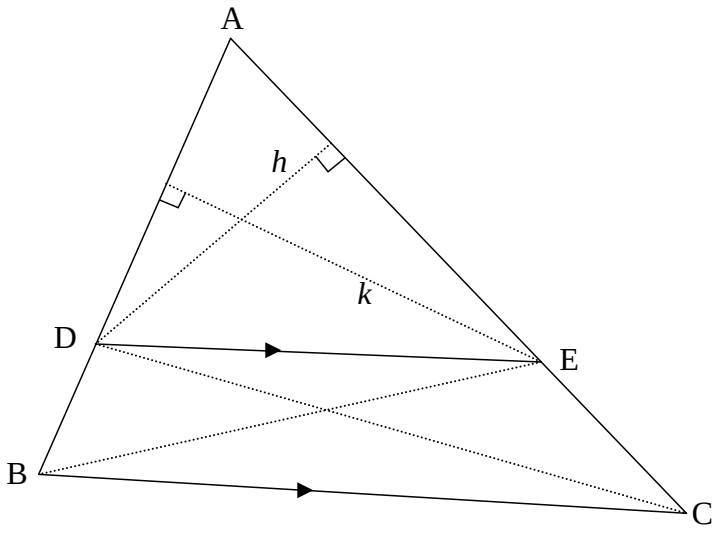


10.2.1	tangent-chord theorem / raaklyn-koordstelling	✓ R	(1)
10.2.2	In ΔABC and ΔADB: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] ∴ ΔABC ||| ΔADB [∠ ; ∠ ; ∠] OR In ΔABC and ΔADB: $\hat{A}_1 = \hat{A}_1$ [common/gemeenskaplik] $\hat{B}_1 = \hat{D}_1$ [proven/bewys in 10.2.1] $B\hat{C}A = \hat{B}_2$ [∠s of Δ = 180°] ∴ ΔABC ||| ΔADB	✓S ✓ S ✓ R ✓S ✓ S ✓ R	(3)

10.2.3	$\hat{E}_2 = \hat{F}_1$ [verwiss ∠e/alternate ∠s ; EA || GF] $\hat{F}_1 = \hat{D}_2$ [ext ∠ of cyc quad DGFC/buite ∠ v kdvh DGFC] $\therefore \hat{E}_2 = \hat{D}_2$	✓ S ✓R ✓ S ✓R	(4)
10.2.4	In ΔAEC and ΔADE: $\hat{A}_2 = \hat{A}_2$ [common/gemeenskaplik] $\hat{E}_2 = \hat{D}_2$ [proven/bewys in 10.2.3] $\therefore$ ΔAEC ||| ΔADE [∠ ; ∠ ; ∠] $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$ OR In ΔAEC and ΔADE: $\hat{A}_2 = \hat{A}_2$ [common/gemeenskaplik] $\hat{E}_2 = \hat{D}_2$ [proven/bewys in 10.2.3] $A\hat{C}E = \hat{G}_1$ [∠s of Δ = 180° OR ext ∠ of cyc quad DGFC/ buite ∠ v kdvh DGFC] $\therefore$ ΔAEC ||| ΔADE $\therefore \frac{AE}{AD} = \frac{AC}{AE}$ $\therefore AE^2 = AD \times AC$	✓S ✓ S ✓ R ✓ S ✓S ✓ S ✓ R ✓ S	(4)
10.2.5	$\frac{AB}{AD} = \frac{AC}{AB}$ [ΔABC ||| ΔADB] $AB^2 = AD \times AC$ $= AE^2$ [from 10.2.4] $\therefore AB = AE$	✓S ✓S ✓S	(3)
			[16]

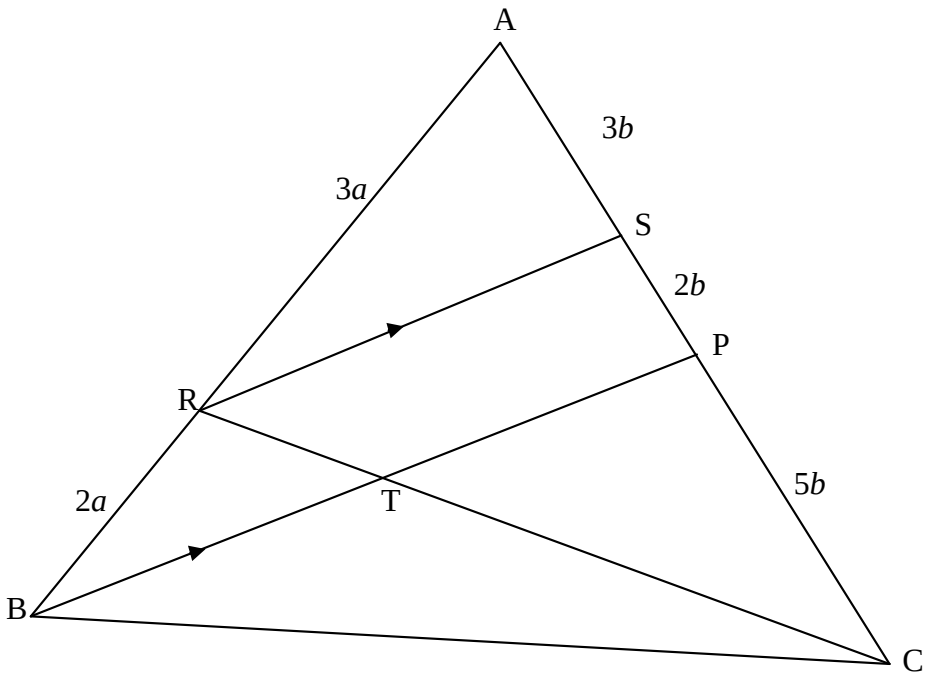
QUESTION/ VRAAG 11

11.1



	Construction: Connect DC and BE and draw the altitudes k and h Konstruksie: Verbind DC en BE en trek hoogtelyne k en h.	✓ constr/ konstr (on sketch or wording/ op skets of beskrywing)	
	$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\frac{1}{2} \times AD \times k}{\frac{1}{2} \times BD \times k} = \frac{AD}{BD}$ $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta DEC} = \frac{\frac{1}{2} \times AE \times h}{\frac{1}{2} \times EC \times h} = \frac{AE}{EC}$ but/maar: Area ΔBDE = Area ΔDEC [DE common base and DE || BC/ DE gemeensk basis en DE || BC] $\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BDE} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta DEC}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓S ✓S ✓S ✓R ✓conclusion	(6)

11.2



11.2.1	$\frac{AS}{SP}=\frac{AR}{RB}$ [line || side of Δ OR RS || BP] $=\frac{3}{2}$ $\frac{AS}{SC}=\frac{3}{7}$ [AP = PC]	✓S ✓R ✓answer ✓S	(4)
11.2.2	$\frac{RT}{TC}=\frac{SP}{PC}$ [line || side of Δ OR RS || TP] $=\frac{2}{5}$	✓S ✓R ✓answer	(3)
11.2.3	$\frac{\text{Area } \Delta TPC}{\text{Area } \Delta RSC}=\frac{\frac{1}{2}TC.PC.\sin T\hat{C}P}{\frac{1}{2}RC.SC.\sin T\hat{C}P}$ $=\frac{TC}{RC}\times\frac{PC}{SC}$ $=\frac{2}{7}\times\frac{5}{7}$ $=\frac{10}{49}$	✓Correct subst into area rule ✓ $\frac{2}{7}$ ✓ $\frac{5}{7}$ ✓answer	(4)
			[17]

Total/Totaal: [150]