GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION/

VOORBEREIDENDE EKSAMEN

2016

MEMORANDUM

MATHEMATICS (SECOND PAPER) /
***WISKUNDE (TWEEDE VRAESTEL)* (10612)**

21 pages / *bladsye*

GAUTENG DEPARTMENT OF EDUCATION /

GAUTENGSE DEPARTEMENT VAN ONDERWYS

PREPARATORY EXAMINATION / *VOORBEREIDENDE EKSAMEN*

MATHEMATICS / *WISKUNDE*
(Second Paper / *Tweede Vraestel*)

MEMORANDUM

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

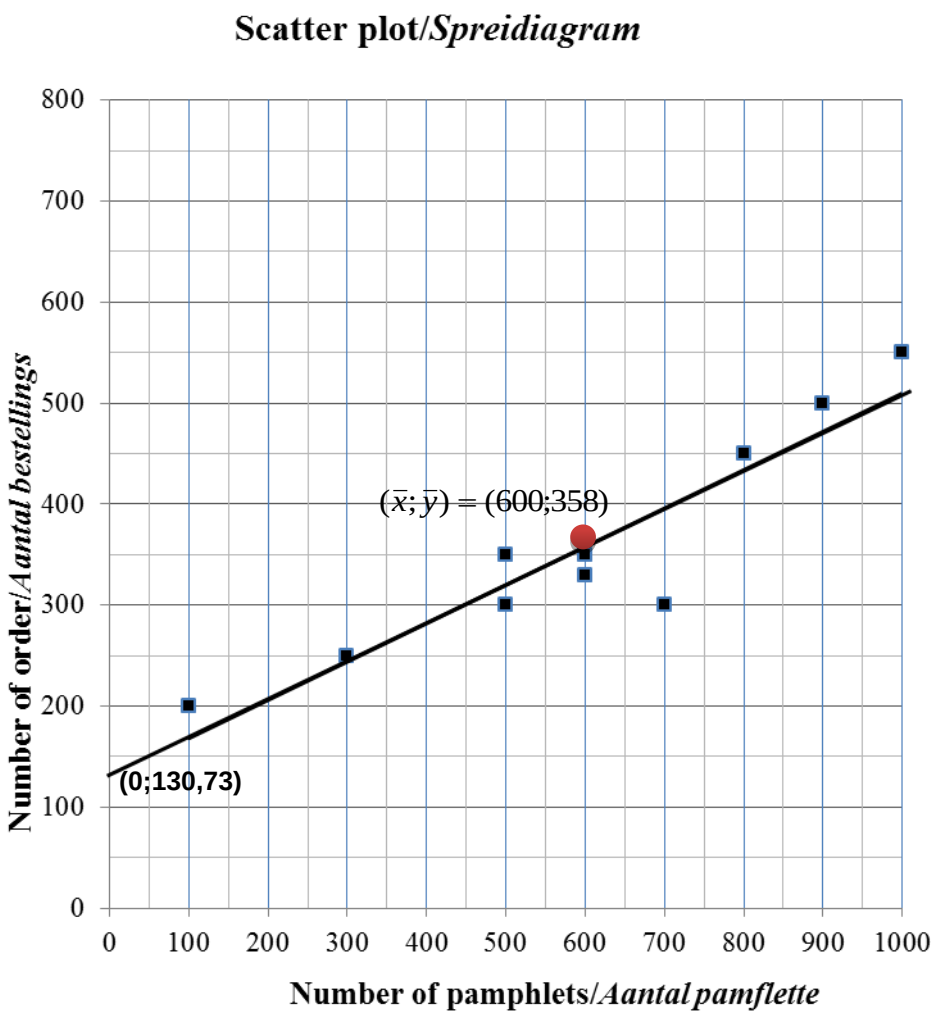
LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

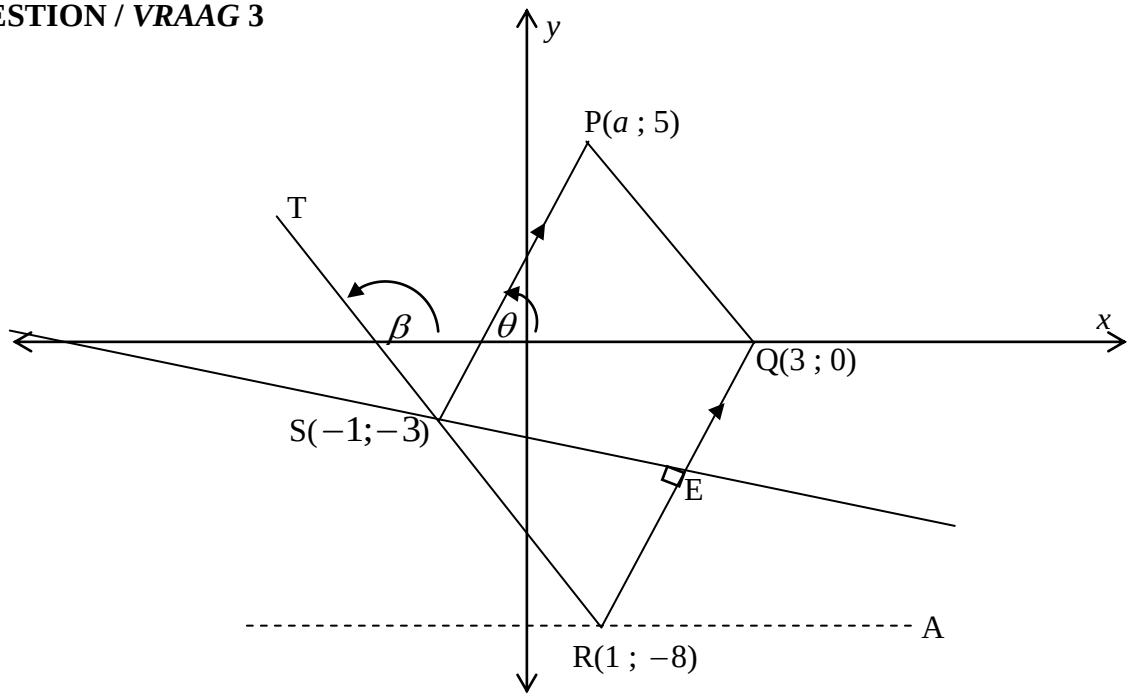
QUESTION / *VRAAG* 1

1.1.1	2015	✓2015 (1)
1.1.2	standard deviation/*standaardafwyking* = 65,18	✓65,18 (2)
1.1.3	$\bar{x} = 164$ (164 – 65,18 ; 164+65,18) = (98,82 ; 229,18) $\frac{8}{12} \times 100 = 66{,}67\%$	✓ $\bar{x} = 164$ ✓ (98,82 ; 229,18) ✓ 66,67% (3)
1.2.1	$220 - 26$ $= 194$	✓194 (1)
1.2.2	skewed to the right /*skeef na regs* **OR / *OF*** positively skewed / *positief skeef*	✓positively/skewed to the right *positief skeef/ skeef na regs* (1)
1.2.3	(a) the mean will increase / *die gemiddeld sal verhoog* (b) the median will stay the same / *die mediaan sal dieselfde bly*	✓mean increases ✓median the same (2)
1.2.4	$Q_1 = 64$ which means 25% of the data lies to the left of Q_1 / 25% *van die data lê links van* Q_1 $25\% \times 12 \text{ years} = 3$ for 3 years /*vir 3 jaar*	✓25% ✓3 years (2) **[12]**

QUESTION / *VRAAG* 2

2.1		no marks: 0 – 3 points correctly ✓plotting 4 – 6 points correctly ✓✓plotting 7 – 9 points correctly *geen punte:* *0 – 3 punte korrek* ✓ *stip 4 – 6 pte korrek* ✓✓*stip 7 –9 pte korrek* (2)
2.2	$y = 0{,}38x + 130{,}73$	✓0,38 ✓130,73 (2)
2.3	*y*-intercept at/ *y-afsnit by* (0 ; 130,73) point $(\bar{x};\bar{y}) = (600;358)$ refer to answer book / *verwys na antwoordboek*	✓(0;130,73) ✓ (600;358) (2)
2.4	$\hat{y}$ = 130,73+0,38(200) = 206,73 ≈ 206 orders/*bestellings* **OR / *OF*** Calculator usage / *sakrekenaar gebruik* $\hat{y}$ = 206,48 = 206 orders/*bestellings*	✓substitution ✓206 correct rounding (accept 207) (2)
		[8]

QUESTION / *VRAAG* 3



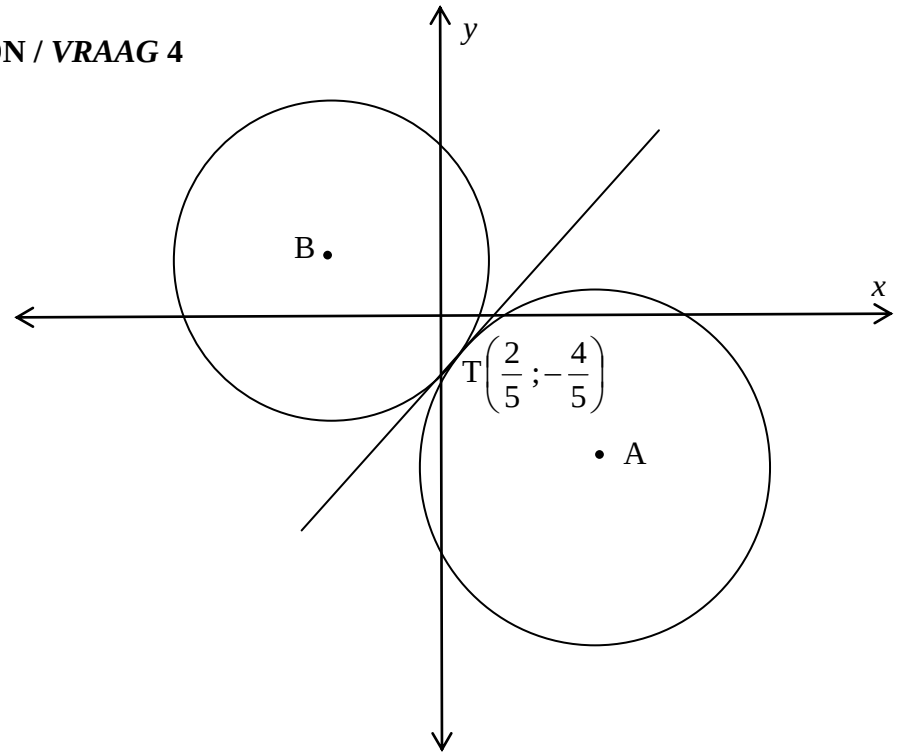
3.1	$QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}^2$ $= \sqrt{(3-1)^2 + (0+8)^2}$ **OR /** $\sqrt{(1-3)^2 + (-8-0)^2}$ $= \sqrt{4+64}$ ***OF*** $= \sqrt{68}$ $= 2\sqrt{17}$ $= 8{,}25$	✓ correct sub into distance formula / *korrekte verv. in afstandsformule* ✓ $2\sqrt{17}$ or 8,25 (2)
3.2	$m_{QR} = \frac{0-(-8)}{3-1}$ **OR / *OF*** $\frac{-8-(0)}{1-3}$ $= \frac{8}{2}$ $= \frac{-8}{-2}$ $= 4$ $= 4$	✓ subst into formula *verv. in formule* ✓4 (2)
3.3	$E\left(\frac{35}{17}; y\right)$ $m_{QR} = 4$ $\frac{y-0}{\frac{35}{17}-3} = 4$ **OR / *OF*** $\frac{y-(-8)}{\frac{35}{17}-1} = 4$ $y = -\frac{64}{17}$ **OR / *OF***	✓ $\frac{y-0}{\frac{35}{17}-3} = 4$ ✓ or $\frac{y-(-8)}{\frac{35}{17}-1} = 4$ $y = -\frac{64}{17}$ (2) **OR / *OF***

	Equation RQ: $y-0=4(x-3)$ **OR / *OF*** $y+8=4(x-1)$ $y=4x-12$ $y=4x-12$ $=4\left(\frac{35}{17}\right)-12$ $=-\frac{64}{17}$	✓ $y=4x-12$ ✓ $y=-\frac{64}{17}$ (2)
3.4	SE ⊥ QR $m_{SE}=-\frac{1}{4}$ **OR / *OF*** $y=-\frac{1}{4}x+c$ $y-y_1=m(x-x_1)$ $-3=-\frac{1}{4}(-1)+c$ $y-(-3)=-\frac{1}{4}(x-(-1))$ $c=-\frac{13}{4}$ $y=-\frac{1}{4}x-\frac{13}{4}$ $y=-\frac{1}{4}x-\frac{13}{4}$ **OR / *OF*** $m_{SE}=-\frac{1}{4}$ **OR / *OF*** $y=-\frac{1}{4}x+c$ $y-y_1=m(x-x_1)$ $-\frac{64}{17}=-\frac{1}{4}\left(\frac{35}{17}\right)+c$ $y-\left(-\frac{64}{17}\right)=-\frac{1}{4}\left(x-\frac{35}{17}\right)$ $c=-\frac{13}{4}$ $y=-\frac{1}{4}x-\frac{13}{4}$ $y=-\frac{1}{4}x-\frac{13}{4}$ This answer could also be left as $4y=-x-13$ **OR / *OF*** $4y+x+13=0$	✓ $m_{SE}=-\frac{1}{4}$ ✓subst / *verv.* (–1 ;–3) ✓answer / *antw.* **OR / *OF*** ✓ $m_{SE}=-\frac{1}{4}$ (3) ✓subst / *verv.* $\left(\frac{35}{17};-\frac{64}{17}\right)$ ✓answer / *antw.* (3)
3.5	For parallel lines: / *Vir parallellyne:* $m_{PS}=m_{QR}$ $\frac{5-(-3)}{a-(-1)}=4$ $8=4(a+1)$ $8=4a+4$ $4a=4$ $a=1$ Answer only: FULL marks. Translation can be used to get the answer.	✓ $\frac{5-(-3)}{a-(-1)}$ ✓ 4 ✓ $a=1$ (3)
3.6.1	$m_{PS}=4$ $\tan\theta=4$ $\theta=75{,}96^{\circ}$	✓ $\tan\theta=4$ ✓ $75{,}96^{\circ}$ (2)

3.6.2	$m_{SR} = \frac{-8-(-3)}{1-(-1)}$ $= -\frac{5}{2}$ $\tan\beta = -\frac{5}{2}$ $= 180^o - 68{,}20^o$ $= 111{,}80^o$ $T\hat{S}P = 111{,}80^o - 75{,}96^o$ ext. angle of a triangle / *buite hoek van 'n driehoek* $= 35{,}84^o$ $\hat{R} = 35{,}84^o$ corresponding angles / *ooreenstemmende hoeke* PS// RQ **OR / *OF*** Draw a line RA parallel to *x*-axis through R / *Trek 'n lyn RA parallel aan die x-as deur R* $m_{QR} = m_{PS} = 4$ $\tan Q\hat{R}A = 4$ $Q\hat{R}A = 75{,}96°$ $m_{SR} = \frac{-8-(-3)}{1-(-1)}$ $= -\frac{5}{2}$ $\tan S\hat{R}A = 180° - 68{,}20°$ $= 111{,}80°$ $S\hat{R}Q = 111{,}80° - 75{,}96°$ $= 35{,}84°$	✓ $-\frac{5}{2}$ ✓ $\tan\beta = -\frac{5}{2}$ ✓ $111{,}80^o$ ✓ $35{,}84^o$ ✓ $\hat{R} = 35{,}84^o$ (5) **OR / *OF*** ✓ 4 ✓ $75{,}96°$ ✓ $-\frac{5}{2}$ ✓ $111{,}80^o$ ✓ $35{,}84^o$ (5)
3.7	SR will be the diameter of the circle (line subt 90º angle) / $SR = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-1-1)^2+(-3+8)^2}$ $= \sqrt{4+25}$ $= \sqrt{29}$ *radius of circle / radius van sirkel* $= \frac{\sqrt{29}}{2}$ or 2,69 **OR / *OF***	✓ subst in distance formula / *verv. in afstandsformule* ✓ $\sqrt{29}$ ✓ $\frac{\sqrt{29}}{2}$ or 2,69 (3) **OR / *OF***

	SR will be the diameter of the circle (line subt 90º angle)	
	Midpoint of SR: / *Middelpunt van SR:* $\left(\frac{1-1}{2};\frac{-8-3}{2}\right)$ $=\left(0;-\frac{11}{2}\right)$	✓$\left(0;-\frac{11}{2}\right)$
	Radius of the circle SRE / *Radius van die sirkel SRE*: $\sqrt{(1-0)^2+\left(-8+\frac{11}{2}\right)^2}$ **OR / *OF*** $\sqrt{(-1-0)^2+\left(-3+\frac{11}{2}\right)^2}$	✓subst in distance formula / *verv. in afstandsform*
	RE: $=\sqrt{1+\frac{25}{4}}$ $=\frac{\sqrt{29}}{2}$ **OR / *OF*** 2,69	✓$\frac{\sqrt{29}}{2}$ or 2,69 (3)
		[22]

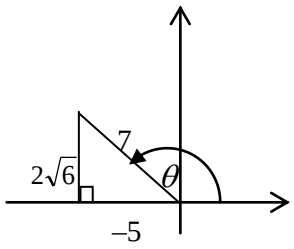
QUESTION / *VRAAG* 4



4.1	A(2 ; –2)	✓ (2 : –2) (1)
4.2.1	$x^2+y^2+4x-2y+p=0$ $x^2+4x+(2)^2+y^2-2y+(-1)^2=(2)^2+(-1)^2-p$ $(x+2)^2+(y-1)^2=5-p$ B(–2 ; 1)	✓–2 ✓1 (2)
4.2.2	$r^2=5-p$ $r=\sqrt{5-p}$	✓5 – p ✓ $r=\sqrt{5-p}$ (2)
4.2.3	$AB=\sqrt{(-2-2)^2+(1+2)^2}$ $=\sqrt{16+9}$ $=\sqrt{25}$ $=5$ **OR / *OF*** $AB=\sqrt{(2+2)^2+(-2-1)^2}$	✓subst. / *verv.* (2 ; –2) & (–2 ; 1) into distance formula / *afstandsformule* ✓ 5 (2)
4.2.4	AB = R + r $5=2+\sqrt{5-p}$ $3=\sqrt{5-p}$ $9=5-p$ $p=-4$	✓ $R+r=2+\sqrt{5-p}$ ✓ $5=2+\sqrt{5-p}$ ✓ $p=-4$ (3)

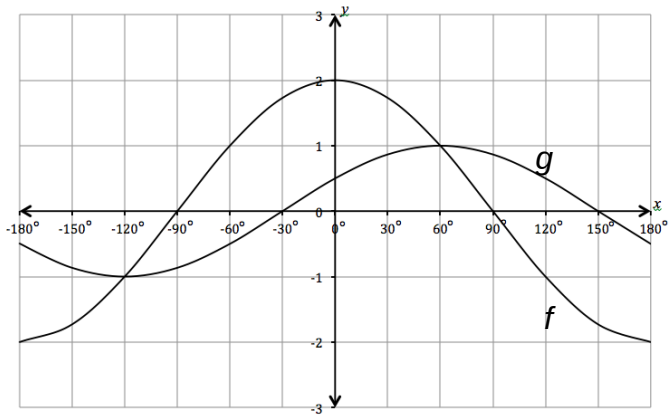
4.3	$m_{AT} = \dfrac{-2+\frac{4}{5}}{2-\frac{2}{5}}$ $= \dfrac{-\frac{6}{5}}{\frac{8}{5}}$ $= -\frac{3}{4}$ **OR / *OF*** $m_{BT} = \dfrac{1+\frac{4}{5}}{-2-\frac{2}{5}}$ $= \dfrac{\frac{9}{5}}{-\frac{12}{5}}$ $= -\frac{3}{4}$	✓ sub (2 ; –2) and $\left(\frac{2}{5};-\frac{4}{5}\right)$ **OR / *OF*** sub / *verv.* (–2;1) and $\left(\frac{2}{5};-\frac{4}{5}\right)$ into gradient / formula / *in gradiënt formula* ✓ $-\frac{3}{4}$
	$m_{AB} \times m_{tangent} = -1$ $m_{tang} = \frac{4}{3}$ $y+\frac{4}{5} = \frac{4}{3}\left(x-\frac{2}{5}\right)$ $y+\frac{4}{5} = \frac{4}{3}x-\frac{8}{15}$ $y = \frac{4}{3}x-\frac{4}{3}$	✓ $m_{tang} = \frac{4}{3}$ ✓ sub $\left(\frac{2}{5};-\frac{4}{5}\right)$ ✓ $y = \frac{4}{3}x-\frac{4}{3}$ (5)
	OR / *OF* A: $x^2-4x+4+y^2+4y+4=4$ $x^2-4x+y^2+4y=-4$ B: $x^2+4x+y^2-2y=4$ A – B: $-8x+6y=-8$ $y = \frac{4}{3}x-\frac{4}{3}$	**OR / *OF*** ✓ $x^2-4x+4+y^2+4y+4=4$ ✓ $x^2-4x+y^2+4y=-4$ ✓ $-8x+6y=-8$ ✓✓ $y = \frac{4}{3}x-\frac{4}{3}$ (5) **[15]**

QUESTION / *VRAAG* 5

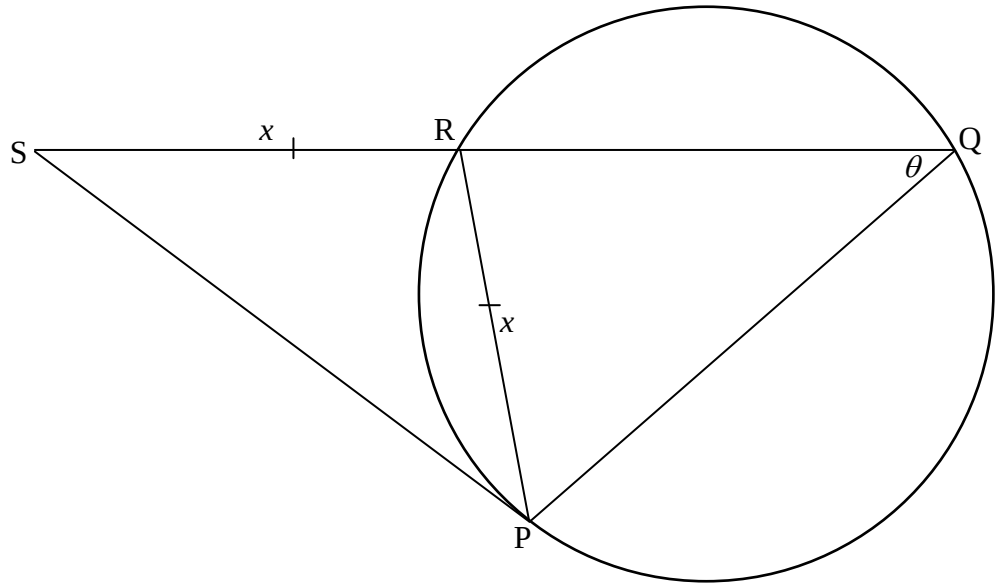
5.1.1	$\tan\theta = -\frac{2\sqrt{6}}{5}$  $r^2 = (2\sqrt{6})^2 + (-5)^2$ Pyth $= 49$ $\therefore r = 7$ $\sin\theta = \frac{2\sqrt{6}}{7}$	✓r = 7 ✓ $\sin\theta = \frac{2\sqrt{6}}{7}$ (2)
5.1.2	$\cos\theta = -\frac{5}{7}$	✓ $\cos\theta = -\frac{5}{7}$ (1)
5.1.3	$\frac{14\cos\theta + 7\sqrt{6}\sin\theta}{\cos(-240^\circ).\tan 225^\circ}$ $= \frac{14\left(-\frac{5}{7}\right) + 7\sqrt{6}\left(\frac{2\sqrt{6}}{7}\right)}{(\cos 240^\circ)(\tan 225^\circ)}$ $= \frac{-10+12}{(-\cos 60^\circ)(\tan 45^\circ)}$ $= \frac{2}{\left(-\frac{1}{2}\right)(1)}$ $= -4$	✓–cos60° ✓tan45° ✓ $-\frac{1}{2}$ and 1 ✓– 4 (4)
5.2	$\frac{\cos\theta - \cos 2\theta + 2}{3\sin\theta - \sin 2\theta}$ $= \frac{\cos\theta - (2\cos^2\theta - 1) + 2}{3\sin\theta - 2\sin\theta\cos\theta}$ $= \frac{\cos\theta - 2\cos^2\theta + 1 + 2}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{3 + \cos\theta - 2\cos^2\theta}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{(3 - 2\cos\theta)(1 + \cos\theta)}{\sin\theta(3 - 2\cos\theta)}$ $= \frac{1 + \cos\theta}{\sin\theta}$ = RHS	✓ $2\cos^2\theta - 1$ ✓ $2\sin\theta.\cos\theta$ ✓ $3 + \cos\theta - 2\cos^2\theta$ ✓ $(3 - 2\cos\theta)(1 + \cos\theta)$ ✓ $\frac{1 + \cos\theta}{\sin\theta}$ (5)

5.3	$\sin\theta\sin\frac{3\theta}{2}+\cos\frac{3\theta}{2}\cos\theta=-\frac{\sqrt{3}}{2}$ $\cos\frac{3\theta}{2}\cos\theta+\sin\frac{3\theta}{2}\sin\theta=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{3\theta}{2}-\theta\right)=-\frac{\sqrt{3}}{2}$ $\cos\left(\frac{\theta}{2}\right)=-\frac{\sqrt{3}}{2}$ $\frac{\theta}{2}=150^\circ+k.360^\circ\ k\in Z$ or/*of* $\frac{\theta}{2}=-150^\circ+k.360^\circ\ k\in Z$ $\theta=300^\circ+k.720^\circ\ k\in Z$ $\theta=-300^\circ+k.720^\circ\ k\in Z$ **OR / *OF*** *ref* $\angle=30^\circ$ $\frac{\theta}{2}=180^\circ-30^\circ+k.360^\circ$ or / *of* $\frac{\theta}{2}=180^\circ+30^\circ+k.360^\circ;k\in Z$ $\theta=300^\circ+k.720^\circ$ or / *of* $\theta=420^\circ+k.720^\circ$	✓ $\cos\left(\frac{3\theta}{2}-\theta\right)$ ✓ $\frac{\theta}{2}=\pm150^\circ+k.360^\circ$ ✓ $\theta=300^\circ+k.720^\circ$ ✓ $\theta=-300^\circ+k.720^\circ$ (4) **OR / *OF*** ✓ $\frac{\theta}{2}=180^\circ\pm30^\circ+k.360^\circ$ ✓ $300^\circ+k.720^\circ$ ✓ $420^\circ+k.720^\circ$ (4)
5.4.1	Maximum value is 1 and minimum value is –1 / *Maksimum waarde is 1 en mimimum waarde –1*	✓answer /*antw.* (1)
5.4.2	Range of both graphs is $-1\le y\le 1$ $\sin\theta.\cos\beta=-1$ $\sin\theta=1$ and / *en* $\cos\beta=-1$ or/ *of* $\sin\theta=-1$ and / *en* $\cos\beta=1$ $\theta=90^\circ$ and / *en* $\beta=-180^\circ$ $\theta=270^\circ$ and / *en* $\beta=0^\circ$	✓ $\theta=90^\circ$ ✓ $\beta=-180^\circ$ ✓ $\theta=270^\circ$ ✓ $\beta=0^\circ$ (4) **[21]**

QUESTION / *VRAAG* 6

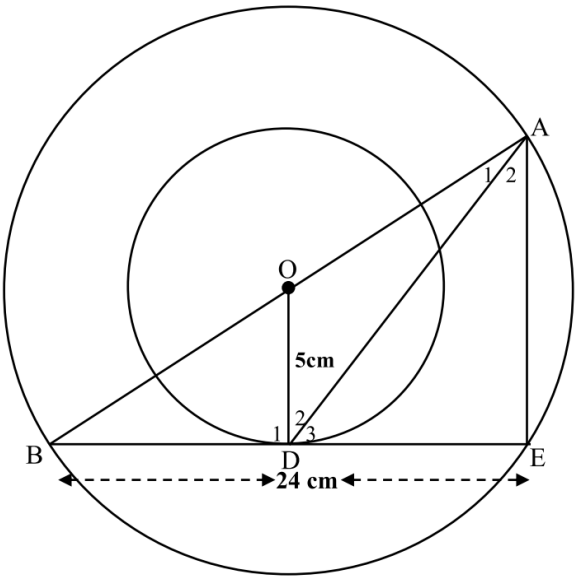
6.1	$2\cos x = \sin(x+30^\circ)$ $2\cos x = \sin x\cos 30^\circ + \cos x\sin 30^\circ$ $2\cos x = \sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right)$ $4\cos x = \sqrt{3}\sin x + \cos x$ $3\cos x = \sqrt{3}\sin x$	✓ $\sin x\cos 30^\circ + \cos x\sin 30^\circ$ ✓ $\frac{\sqrt{3}}{2}$ and / *en* $\frac{1}{2}$ ✓ $4\cos x = \sqrt{3}\sin x + \cos x$ (3)
6.2	$\frac{3\cos x}{\cos x} = \frac{\sqrt{3}\sin x}{\cos x}$ $\tan x = \sqrt{3}$ ref $\angle = 60^\circ$ $x = 60^\circ + k360^\circ$ **OR / *OF*** $x = 240^\circ + k.360^\circ; k \in Z$ $\therefore x = \{-120^\circ; 60^\circ\}$	✓ $\tan x = \sqrt{3}$ ✓ ref angle / *verw. hoek* ✓ -120° ✓ 60° (4)
6.3.1	$y \in [-2;2]$ **OR / *OF*** $-2 \le y \le 2$	✓ $y \in [-2;2]$ (1)
6.3.2		✓ *x*-intercepts ✓ *y*-intercept ✓ shape (3)
6.3.3	360°	✓ 360° (1)
6.3.4	$-120^\circ < x < 60^\circ$ **OR / *OF*** $x \in (-120^\circ; 60^\circ)$	✓ $-120^\circ < x$ ✓ $x < 60^\circ$ (2)
6.3.5	$-30^\circ < x < 90^\circ$ **OR / *OF*** $x \in (-30^\circ; 90^\circ)$ **Brackets/ Inequality signs have to be correct.**	✓ $-30^\circ < x$ ✓ $x < 90^\circ$ (2) **[16]**

QUESTION / *VRAAG* 7



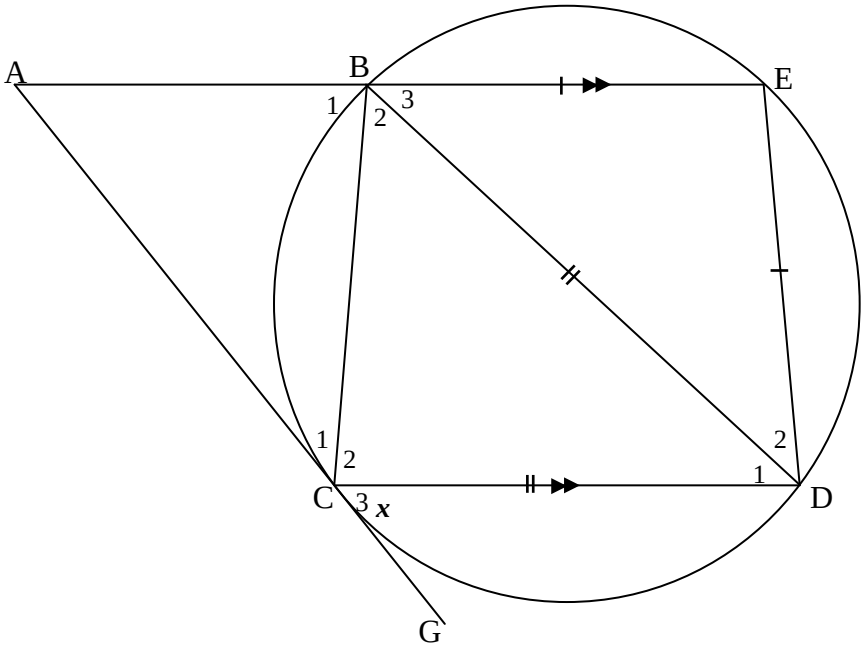
7.	$\hat{SPR} = \theta$ [tan-chord / *raaklyn-koord*] $\hat{R} = 180° - 2\theta$ [sum $\angle^s$ of Δ / *som* $\angle^e$ *van* Δ] $PS^2 = x^2 + x^2 - 2x^2\cos(180° - 2\theta)$ $= 2x^2 + 2x^2\cos 2\theta$ $= 2x^2(1 + \cos 2\theta)$ $= 2x^2(1 + 2\cos^2\theta - 1)$ $= 2x^2(2\cos^2\theta)$ $= 4x^2\cos^2\theta$ $PS = 2x\cos\theta$ **OR / *OF*** $PS^2 = x^2 + x^2 - 2x^2\cos(180° - 2\theta)$ $= 2x^2 + 2x^2\cos 2\theta$ $= 2x^2(1 + \cos 2\theta)$ $= 2x^2(1 + 1 - 2\sin^2\theta)$ $= 2x^2(2 - 2\sin^2\theta)$ $= 4x^2(1 - \sin^2\theta)$ $= 4x^2\cos^2\theta$ $PS = 2x\cos\theta$	✓ $\hat{SPR} = \theta$ ✓ $\hat{R} = 180° - 2\theta$ ✓ $x^2 + x^2 - 2x^2\cos(180° - 2\theta)$ ✓ $+ 2x^2\cos 2\theta$ ✓ $2\cos^2\theta - 1$ ✓ $4x^2\cos^2\theta$ (6) **OR / *OF*** ✓ $x^2 + x^2 - 2x^2\cos(180° - 2\theta)$ ✓ $+ 2x^2\cos 2\theta$ ✓ $1 - 2\sin^2\theta$ ✓ $4x^2\cos^2\theta$ (6) **[6]**

QUESTION / *VRAAG* 8



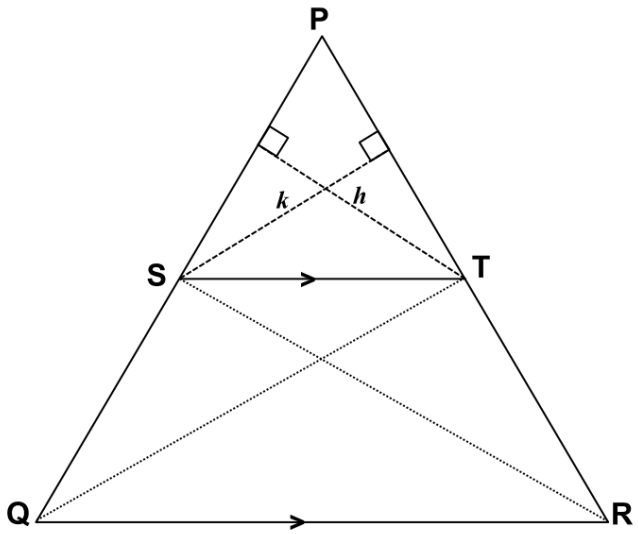
8.1	bisects / *halveer*		✓ answ/*antw.* (1)
8.2.1	$OD \perp BE \left(\hat{D}_1 = \hat{D}_2 + \hat{D}_3 = 90^o\right)$ $\therefore BD = DE = 12cm$ $OB^2 = 12^2 + 5^2$ $= 169$ $OB = 13cm$	[tan ⊥ radius / raaklyn ⊥ radius] [line from centre ⊥ to chord / *Loodlyn uit midpt. ⊙ na koord*] [Pyth; $\hat{D}_1 = 90°$]	✓ S/R ✓ S/R ✓ $OB^2 = 12^2 + 5$ ✓ OB = 13 (4)
8.2.2	$\hat{E} = 90°$	[∠ in semi ⊙/*∠ in halwe ⊙*] OR [corresp ∠s; DO ‖ AE / *ooreenk. ∠;* DO ‖ AE]	✓R (1)
8.2.3	OD‖ AE and/ *en* BD = DE $AE = 2OD$ $= 10$ **OR / *OF*** $OB = OA$ $AB = 26$ In ΔABE $AE^2 = 26^2 - 24^2$ $= 100$ $AE = 10$	[Midpt Theorem /*Midpt.-stelling*] [radii] [Pyth; $\hat{E} = 90°$]	✓S/R ✓ AE = 2OD ✓AE = 10 (3) **OR / *OF*** ✓ AB = 26 ✓ S/R ✓AE = 10 (3)
8.2.4	In ΔADE $AD^2 = 10^2 + 12^2$ $= 244$ $AD = \sqrt{244}$ **OR / *OF*** $2\sqrt{61}$ **OR / *OF*** 15,62	[Pyth; $\hat{E} = 90°$]	✓S/R ✓ $AD = \sqrt{244}$ **OR / *OF*** $2\sqrt{61}$ **OR / *OF*** 15,62 (2) **[11]**

QUESTION / *VRAAG* 9

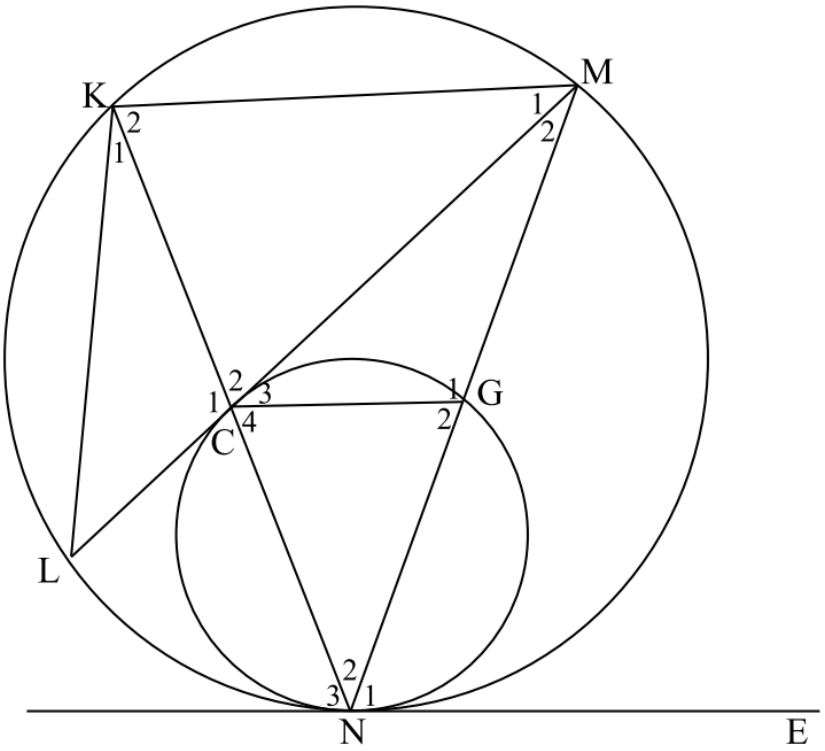


9.1	$x = \hat{C}_3$	[given / *gegee*]	
	$= \hat{B}_2$	[tan chord theorem / *∠tussen raaklyn en koord*]	✓ S ✓R
	$= \hat{C}_2$	[∠s opp equal sides /*∠e teenoor gelyke sye*	✓S/R
	$= \hat{B}_1$	[alt ∠s; BE || CD/*verw.∠e; BE || CD*]	✓S/R
	$= \hat{D}_1 + \hat{D}_2$	[ext ∠ of cyclic quad /*buite ∠van kvh]*	✓S ✓R
	$\hat{C}_3 = \hat{A} = x$	[corresp ∠s; BE || CD / *ooreenk. ∠; BE || CD*]	✓S/R (7)
9.2	$\hat{B}_3 = \hat{D}_1$	[alt ∠s; BE || CD /*verw.∠e; BE || CD*]	✓S/R
	ED = BC	[equal ∠s ; equal chords *gelyke omtr ∠e; gelyke koorde*]	✓S/R (2)
	OR / *OF*		**OR / *OF***
	In Δ BCD and/*en* Δ EDC		
	CD = CD	[common / *gemeenskaplik*]	
	$\hat{C}_2 = E\hat{D}C = x$	[proven in 9.1 / *bewys in* 9.1]	✓S/R for all three angles/ *vir al drie hoeke*
	$\hat{B}_2 = C\hat{E}D$	[∠s in the same seg /*∠e in dies. ⊙ segm.*	
	Δ BCD ≡ Δ EDC	[AAS OR ∠∠S / *HHS OF ∠∠S*]	
	ED = BC	[from ≡ / *uit* ≡]	✓ S/R (2) **[9]**

QUESTION / *VRAAG* 10



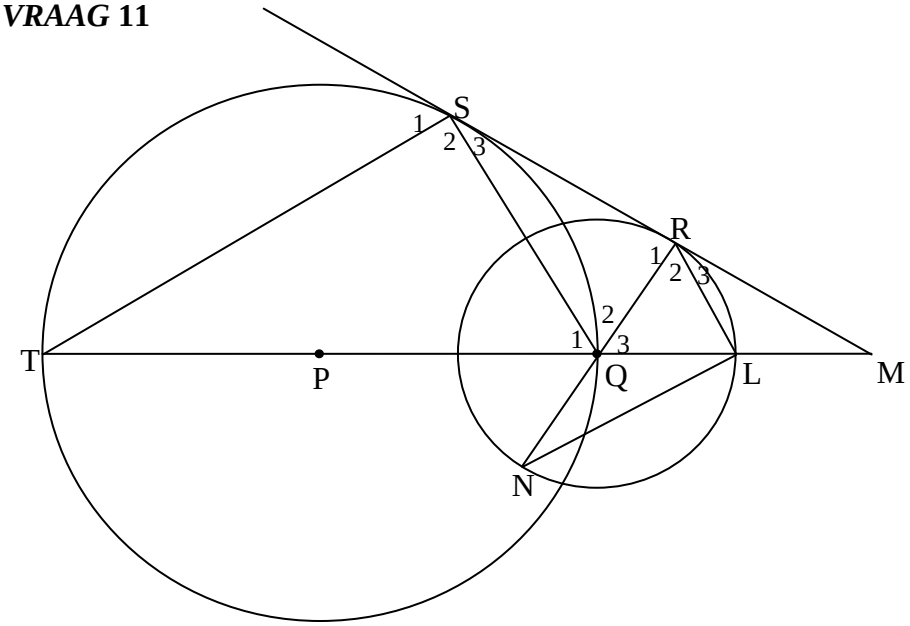
10.1	***Construction***: In ΔPQR, draw height h relative to base PS and the height k relative to base PT. Join QT and RS to form ΔSQT and Δ TRS. $\frac{\text{Area of } \Delta PST}{\text{Area of } \Delta SQT} = \frac{\frac{1}{2} \times PS \times h}{\frac{1}{2} \times SQ \times h} = \frac{PS}{SQ}$ $\frac{\text{Area of } \Delta PTS}{\text{Area of } \Delta TRS} = \frac{\frac{1}{2} \times PT \times k}{\frac{1}{2} \times TR \times k} = \frac{PT}{TR}$ But Area ΔSQT = Area ΔTRS $\frac{\text{Area of } \Delta PST}{\text{Area of } \Delta SQT} = \frac{\text{Area of } \Delta PTS}{\text{Area of } \Delta TRS}$ $\frac{PS}{SQ} = \frac{PT}{TR}$	[same base, equal height **OR** same base, ST ∥ QR /*dies. basis ; gelyke hoogte* **OF** *dies. basis,* ST ∥ QR]	✓ Construction **on sketch**/ *Konstruksie* **op skets** ✓S ✓S ✓S/R ✓S (5)



10.2.1	$\hat{N}_1 = \hat{C}_4$	[tan chord theorem / *∠tussen raaklyn en koord*]	✓S /R
	$\hat{N}_1 = \hat{K}_2$	[tan chord theorem / *∠tussen raaklyn en koord*]	✓S/R
	$\therefore \hat{C}_4 = \hat{K}_2$		
	CG ∥ KM	[corresp ∠s = /*ooreenk.∠e gelyk*]	✓S/R
	$\frac{KC}{KN} = \frac{MG}{MN}$	[line ∥ one side of Δ **OR** prop theorem; CG ∥ KM / *lyn ∥ een sy van Δ*]	✓R (4)
10.2.2	$\hat{C}_4 = \hat{K}_2$	proved	
	$\hat{C}_4 = \hat{G}_2$	[∠s opp equal sides / *∠e teenoor gelyke sye*]	✓ $\hat{C}_4 = \hat{G}_2$ ✓R
	$\therefore \hat{G}_2 = \hat{K}_2$		
	$\therefore$ KMGC is a cyclic quad	[ext ∠ = int opp ∠ / *buite hoek = oorst binne∠*]	✓R (3)
10.2.3	In Δ MCG and/*en* Δ MNC		
	$\hat{M}_2 = \hat{M}_2$	[common / *gemeenskaplik*]	✓S
	$\hat{C}_3 = \hat{N}_2$	[tan chord theorem / *∠tussen raaklyn en koord*]	✓S/R
	$\hat{G}_1 = \hat{C}_3 + \hat{C}_4$	[∠ sum in Δ/*∠ som van Δ*]	
	ΔMCG ||| ΔMNC	[∠∠∠]	✓R (3)

10.2.4	$\frac{MC}{MG}=\frac{MN}{MC}$	[Δ^s ||| / Δ^e |||]	✓S/R
	$MC^2=MG.MN$		
	$\frac{MC^2}{MN^2}=\frac{MG.MN}{MN^2}$		✓S
	$=\frac{MG}{MN}$		✓S
	$\frac{KC}{KN}=\frac{MG}{MN}$	[proved / *bewys*]	✓S
	$\frac{MC^2}{MN^2}=\frac{KC}{KN}$		(4) **[19]**

QUESTION / *VRAAG* 11



11.1	$\hat{Q}_3 = 2\hat{N}$ $\hat{N} = 30^o$ $\hat{N} = \hat{R}_3$ $\hat{R}_3 = 30^o$	[∠ at centre = 2× ∠ at circ / *middelpts ∠ = 2× omtrek ∠*] [tan chord theorem / *∠tussen raaklyn en koord*]	✓ Reason ✓ $\hat{N} = 30^o$ ✓Reason ✓ $\hat{R}_3 = 30^o$ (4)
11.2	$\Delta TSQ \|\|\| \Delta SRQ$ $\frac{TS}{SR} = \frac{SQ}{RQ} = \frac{TQ}{SQ}$ $SQ^2 = TQ.RQ$ $= 4r.r$ $= 4r^2$ $SQ = 2r$ **OR / *OF*** $PQ = 2r$ $\therefore TQ = 4r$ $\frac{SQ}{RQ} = \frac{TQ}{SQ}$ $SQ^2 = TQ.RQ$ $= 4r^2$ $SQ = 2r$	[Δ^s ||| / Δ^e |||]

11.3	$\hat{Q}_1 = \hat{Q}_2$	[Δ^s ||| / Δ^e |||]	
	$\hat{Q}_1 = 60°$	[$\angle^s$ on str line / $\angle^e$ *op rt.lyn*]	✓ $\hat{Q}_1 = 60°$
	$\hat{S}_2 = 90°$	[$\angle$ in semi-circle /$\angle$ *in semi-sirkel*]	✓ $\hat{S}_2 = 90°$
	$\hat{T} = 30°$	[sum $\angle^s$ of Δ /*som* $\angle^e$ *van* Δ]	✓ $\hat{T} = 30°$
	$\hat{S}_3 = 30°$	[tan chord theorem / $\angle$*tussen raaklyn en koord*]	✓S/R
	$T\hat{S}R = 120°$		✓ $T\hat{S}R = 120°$ (4)
	OR / *OF*		**OR / *OF***
	$\sin \hat{S}_3 = \frac{RQ}{SQ}$		
	$= \frac{r}{2r}$		
	$= \frac{1}{2}$		✓ $\sin \hat{S}_3 = \frac{1}{2}$
	$\hat{S}_3 = 30°$		✓ $\hat{S}_3 = 30°$
	$\hat{S}_2 = 90°$	[$\angle$ in semi-circle /$\angle$ *in semi-sirkel*]	✓ S/R
	$T\hat{S}R = \hat{S}_3 + \hat{S}_2$		
	$= 120°$		✓ $T\hat{S}R = 120°$ (4) **[11]**

TOTAL / *TOTAAL*: 150