Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE/*GRADE* 12

SEPTEMBER 2016

MATHEMATICS P2 / *WISKUNDE V2* MEMORANDUM

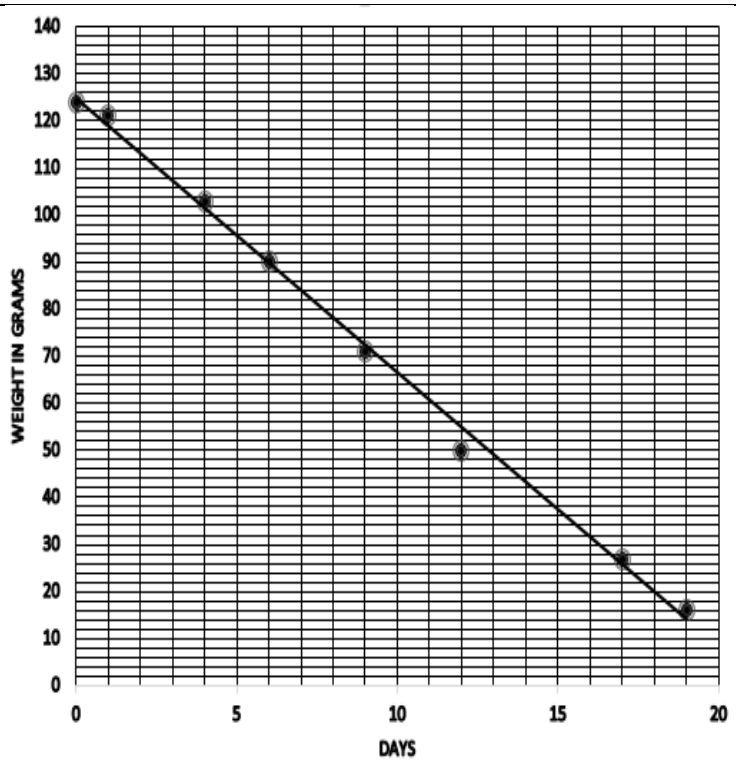
MARKS / *PUNTE*: **150**

This memorandum consists of 16 pages.
Hierdie memorandum bestaan uit 16 bladsye.

QUESTION 1 / *VRAAG 1*

Day/Dag	0	1	4	6	9	12	17	19
Weight / Gewig	124	121	103	90	71	50	27	16

1.1



✓ 2-4 correct points
✓ 5-7 correct points
✓ plotting all points

✓ *2 – 4 punte korrek*
✓ *5 – 7 punte korrek*
✓ *alle punte afgesteek*

(3)

1.2

$a = 124{,}84$
$b = -5{,}83$
$y = 124{,}84 - 5{,}83x$

✓ A
✓ B
✓ equation / *vergelyking*

(3)

1.3

$(x; y) = (8{;}5; 75{,}25)$
y-int 124,84

✓ $(8{;}5; 75{,}25)$ and/*en* *y*-int / *y-afsnit* 124,84
✓ regression line / *regressielyn*

(2)

1.4

$124{,}84 - 5{,}83x = 80$
$5{,}83x = 44{,}84{,}69$
$x = 7{,}69$

On the morning of the 8th day the bar of soap will be less than 80 grams. *Op die oggend van die 8ste dag sal steen seep minder as 80 gram wees.*

✓ substitution / vervanging
✓ answer / *antwoord*

(2)

1.5

$r = -0{,}998$

✓ answer / *antwoord*

(1)

1.6

Very strong negative correlation. *Baie sterk negatiewe korrelasie.*

✓ answer / *antwoord*

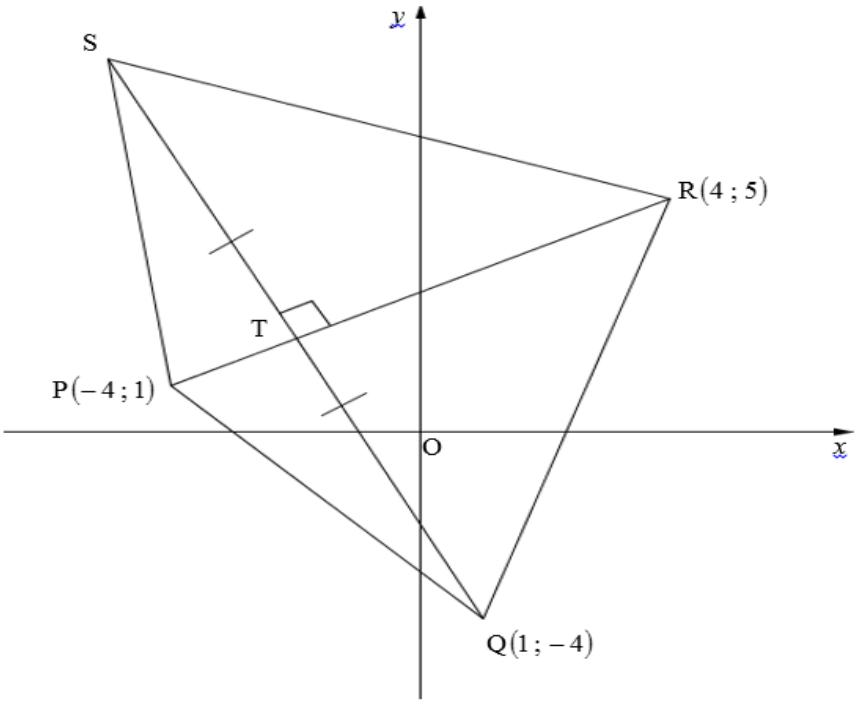
(1)

[12]

QUESTION 2 / *VRAAG 2*

	5 | 7 4 | 3 4 4 3 | 1 1 1 4 6 8 2 | 2 4 8 9 9 1 | 5 5 6 7 0 | 3 5		
2.1	21 learners / *leerders*	✓ answer / *antwoord*	(1)
2.2	3 pages / *bladsye*	✓ answer / *antwoord*	(1)
2.3	$\bar{x} = 28{,}19$	✓✓ answer / *antwoord*	(2)
2.4	$\sigma = 13{,}12$	✓ answer / *antwoord*	(1)
2.5	$(28{,}19 - 13{,}12;\ 28{,}19 + 13{,}12)$ $(15{,}07;41{,}31)$ $\therefore$ 8 learners are outside one standard deviation *8 leerders is buite een standaardafwyking* $\therefore \frac{8}{21} \times 100 = 38{,}10\%$	✓ interval / *interval* ✓ 8 learners / *leerders* ✓ 28,57%	(3)
			[8]

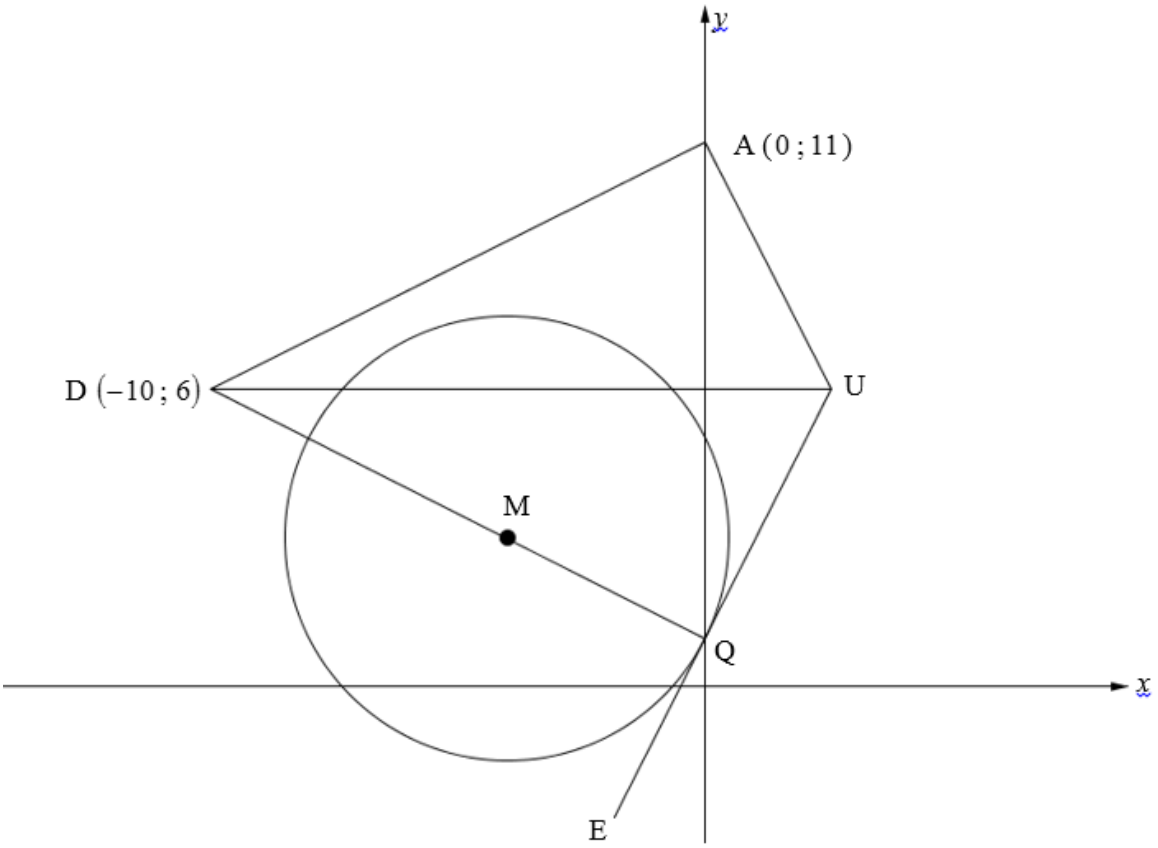
QUESTION 3 / *VRAAG 3*



3.1	$m_{PR} = \frac{5-1}{4+4}$ $= \frac{1}{2}$	✓ subst. P and R into correct formula *verv. van P en R in korrekte formule* ✓ $m_{PR} = \frac{1}{2}$	(2)
3.2	$m_{SQ} = -2$ SP ⊥ PT $y + 4 = -2(x-1)$ $y = -2x - 2$	✓ $m_{SQ} = -2$ ✓ subst. m and Q into correct formula *verv. van m en Q in korrekte formule* ✓ $y = -2x - 2$	(3)
3.3	Equation of PR / *Vergelyking van PR* $y - 1 = \frac{1}{2}(x+4)$ $y = \frac{1}{2}x + 3$ $-2x - 2 = \frac{1}{2}x + 3$ $-4x - 4 = x + 6$ $-5x = 10$ $\therefore x = -2$ $y = -2(-2) - 2 = 2$ **OR/OF**	✓ substituting m and P into equation of a str. line / *vervanging van m en P in die vergelyking van reguitlyn* ✓ equation of PR / *vergelyking van PR* ✓ equating PR and SQ / *gelykstel van PR en SQ* ✓ *x*-value / *x-waarde* ✓ *y*-value / *y-waarde* ✓ substituting m and R into equation of a	

	$y-5=\frac{1}{2}(x-4)$ $2y-10=x-4$ $2y=x+6$ $y=\frac{1}{2}x+3$ $-2x-2=\frac{1}{2}x+3$ $-4x-4=x+6$ $-5x=10$ $\therefore \quad x=-2$ $y=-2(-2)-2=2$	str. line / *vervanging van m en R in die vergelyking van reguitlyn* ✓ equation of PR / *vergelyking van PR* ✓ equating PR and SQ / *gelykstel van PR en SQ* ✓ *x*-value / *x-waarde* ✓ *y*-value / *y-waarde*	(5)
3.4	$-2=\frac{x+1}{2} \qquad 2=\frac{y-4}{2}$ $\therefore x=-5 \qquad$ and/*en* $\qquad y=8$ S (−5 ; 8)	✓ substituting into correct formula *vervanging in korrekte formule* ✓ *x*-value / *x-waarde* ✓ *y*-value / *y-waarde*	(3)
3.5	$SQ=\sqrt{(-5-1)^2+(8+4)^2}$ $=6\sqrt{5}$ $PT=\sqrt{(-2+4)^2+(2-1)^2}$ $=\sqrt{5}$ $\text{Area } \Delta PQS=\frac{1}{2}\times 6\sqrt{5}\times\sqrt{5}$ $=15 \text{ unit}^2$ / *eenhede*^{2} **OR/OF** $\therefore \text{area of } \Delta PQS = ST\times PT$ $=3\sqrt{5}\times\sqrt{5}$ $=15\,\text{unit}^2$ / *eenhede*^{2}	✓ subt. into correct form / *verv. in korrekte formule* ✓ SQ = $6\sqrt{5}$ ✓ PT = $\sqrt{5}$ ✓ Subt into correct form. / *verv. in korrekte formule* ✓ 15 units^2 / eenhede^2 ✓SQ = $6\sqrt{5}$ ✓ST = $3\sqrt{5}$ ✓PT = $\sqrt{5}$ ✓ subst into form / *verv. in formule* ✓ 15 units^2 / eenhede^2	(5)
			[18]

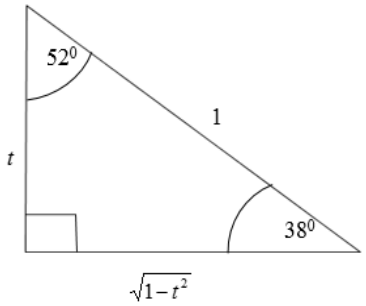
QUESTION 4 / *VRAAG 4*



4.1	$x^2+8x+16+y^2-6y+9=20$ $(x+4)^2+(y-3)^2=20$ $\therefore M(-4\ ;\ 3)$	✓ completing square *kwadraatsvoltooiing* ✓ $(x+4)^2+(y-3)^2=20$ ✓ *x*-coordinate / *x-koördinaat* ✓ *y*-coordinate / *y-koördinaat*	(4)
4.2	$(0+4)^2+(y-3)^2=20$ $(y-3)^2=4$ $y=3\pm 2$ $\therefore y=1$ Q (0; 1)	✓ subst. $x = 0$ into circle equation *verv. x = 0 in sirkel vergelyking* ✓ $(y-3)^2=4$ ✓ $y=1$	(3)
4.3	$m_{radius}=\frac{3-1}{-4-0}=-\frac{1}{2}$ $m_{\tan/rkl}=2$ $y-1=2(x-0)$ [tangent ⊥ radius] *[raaklyn ⊥* $\therefore y=2x+1$	✓ $m_{rad}=-\frac{1}{2}$ ✓ $m_{\tan}=2$ / $m_{raaklyn}=2$ ✓ subst. $m_{\tan/rkl}=2$ and Q into correct form. / *verv.* $m_{\tan/rkl}=2$ *en Q in korrekte formule.* ✓ equation / *vergelyking*	(4)
4.4	$y=6$	✓ answer / *antwoord*	(1)

4.5	$6 = 2x + 1$ $x = \frac{5}{2}$ $\text{U}\left(\frac{5}{2}; 6\right)$	✓ $6 = 2x + 1$ ✓ $x = \frac{5}{2}$	(2)
4.6	$m_{\text{AU}} = \frac{11-6}{0-\frac{5}{2}}$ $= -2$ $m_{\text{AD}} = \frac{6-11}{-10-0}$ $= \frac{1}{2}$ $m_{\text{AU}} \times m_{\text{DA}} = -2 \times \frac{1}{2}$ $= -1$ $\therefore$ AU $\perp$ DA $\therefore$ $\hat{\text{A}} = 90^\circ$ $\text{D}\hat{\text{Q}}\text{U} = 90^\circ$ [tangent $\perp$ radius] / [*raaklyn* $\perp$ *radius*] $\therefore$ QUAD is a cyclic quad.[opp. $\angle^s$ add up to 180º] *QUAD is 'n koordevierhoek [teenoorst. $\angle^e$ se som is 180º]*	✓ $m_{\text{AU}} = -2$ ✓ $m_{\text{AD}} = \frac{1}{2}$ ✓ $m_{\text{AU}} \times m_{\text{DA}} = -1$ ✓ $\hat{A} = 90^\circ$ ✓ $D\hat{Q}U = 90^\circ$ ✓ R	(6)
			[20]

QUESTION 5 / *VRAAG 5*



5.1.1	$\sin(-52°) = -\sin 52°$ $= \sqrt{1-t^2}$	✓ $-\sin 52°$ ✓ ✓answer	(3)
5.1.2	$\cos(2.19°) = \cos 38°$ $2\cos^2 19° - 1 = \cos 38°$ $\cos 19° = \sqrt{\frac{\cos 38° + 1}{2}}$ $\therefore \cos 19° = \sqrt{\frac{\sqrt{1-t^2}+1}{2}}$	✓ $\cos(2.19°) = \cos 38°$ ✓ $\cos(2.19^\circ) = 2\cos^2 19° - 1$ ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord*	(4)
5.2	$\frac{2\cos(180° + x).\sin(180° - x).\sin 74°}{\sin(x + 360°).\sin 37°.\sin 53°.\sin(x - 90°)}$ $= \frac{2(-\cos x).\sin x.\sin 74^\circ}{\sin x.\sin 37°.\cos 37°.(-\cos x)}$ $= \frac{2\sin 74°}{\frac{1}{2}\sin 74°}$ $= 4$ OR/*OF* $\frac{2\cos(180^0 + x).\sin(180^0 - x).\sin 74^0}{\sin(x + 360^0).\sin 37^0.\sin 53^0.\sin(x - 90^0)}$ $= \frac{2(-\cos x).\sin x.\sin(2 \times 37^0)}{\sin x.\sin 37^0.\cos 37^0.(-\cos x)}$ $= \frac{2\sin(2 \times 37^0)}{\sin 37^0.\cos 37^0}$ $= \frac{4\sin 37^0.\cos 37^0}{\sin 37^0.\cos 37^0}$ $= 4$	✓ $-\cos x$ ✓ $\sin x$ ✓ $\sin x$ ✓ $\cos 37°$ ✓ $-\cos x$ ✓ $\frac{1}{2}\sin 74°$ ✓ answer / *antwoord* ✓ $-\cos x$ ✓ $\sin x$ ✓ $\sin x$ ✓ $\cos 37°$ ✓ $-\cos x$ ✓ $2\sin(2 \times 37^0)$ ✓ answer / *antwoord*	(7)

5.3.1	$1-\cos 2x=0$ $\cos 2x=1$ $\therefore \quad 2x=0°+k.360°$ $x=0°+k.180°;k\in Z$	✓ $1-\cos 2x=0$ ✓ $2x=0°+360°.k$ ✓ $x=0°+180°.k;$ ✓ $k\in Z$	(4)
5.3.2	L.H.S/LK $=\dfrac{2\sin x}{2(1-\cos 2x)}$ $=\dfrac{2\sin x}{2-2\cos 2x}$ $=\dfrac{2\sin x}{2-2(1-2\sin^2 x)}$ $=\dfrac{2\sin x}{2-2+2\sin^2 x}$ $=\dfrac{2\sin x}{2\sin^2 x}$ $=\dfrac{1}{\sin x}$	✓ $1-2\sin^2 x$ ✓ removing brackets *verwyder hakies* ✓ $2\sin^2 x$	(3)
			[21]

QUESTION 6 / *VRAAG 6*

6.1

$\sin(x+60^0)=\sin(90^0-2x)$

$\therefore x+60^0=90^0-2x+360^0.k$ or/*of* $x+60^0=180^0-(90^0-2x)+360^0.k$

$3x=30^0+360^0.k$ or/*of* $x=-30^0+360^0.k$

$x=10^0+120^0.k$; $k\in Z$

$\therefore x=-30^0\ ;\ 10^0\ ;\ 130^0$

OR/*OF*

$\cos 2x=\cos(30°-x)$

$2x=30°-x+360°.k$ or $2x=-(30°-x)+360°.k$; $k\in Z$

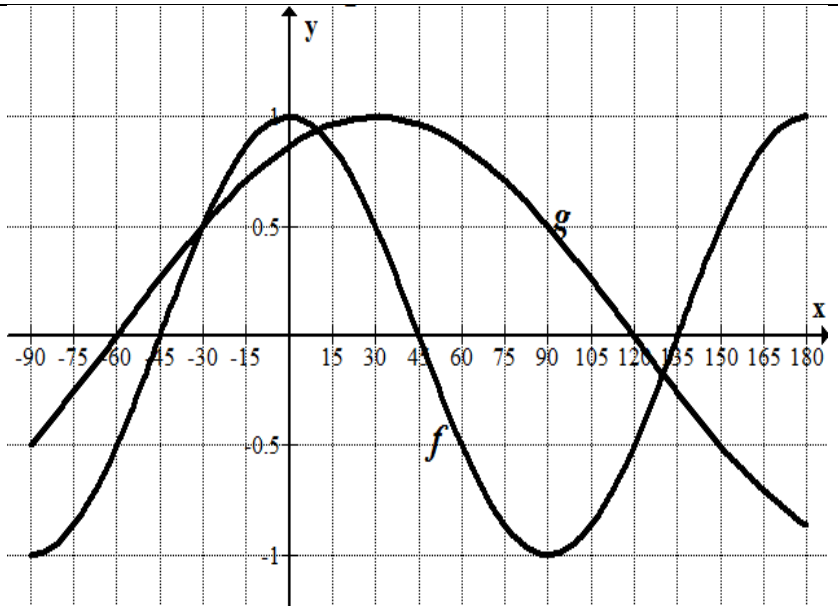
$3x=30°+360°.k$ or $x=-30^{\circ}+360°.k$

$\therefore x=-30°;10°;130°$

✓co-ratio / *ko-funksie*
✓both gen. solns *beide algemene oplossings*
✓-30°
✓10°
✓130°

✓co-ratio / *ko-funksie*
✓both gen. solns *beide algemene oplossings*
✓-30°
✓10°
✓130° (5)

6.2



g:
✓ *x*-intercept. *x-afsnit*
✓ *y*-intercept. *y-afsnit*
✓ shape / *vorm*

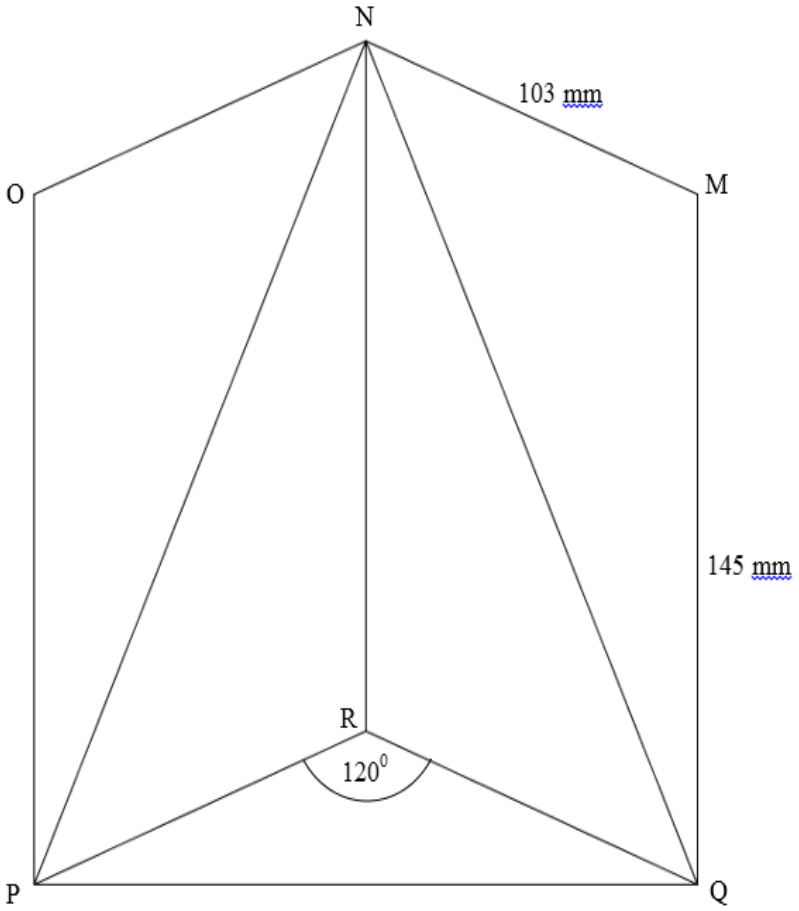
f:
✓ *x*-intercept. *x-afsnit*
✓ *y*-intercept. *y-afsnit*
✓ shape / *vorm* (6)

6.3 240° ✓ answer / *antwoord* (1)

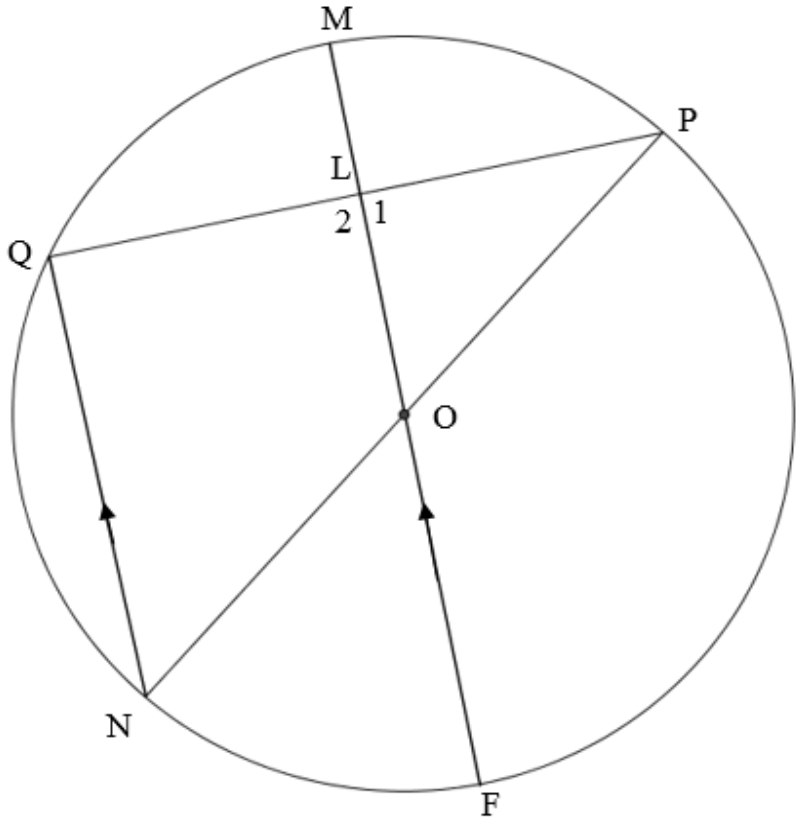
6.4 $h(x)=\cos(2x-90°)-1=\sin 2x-1$ ✓ substitution / vervanging ✓ sin 2*x* – 1 (2)

[14]

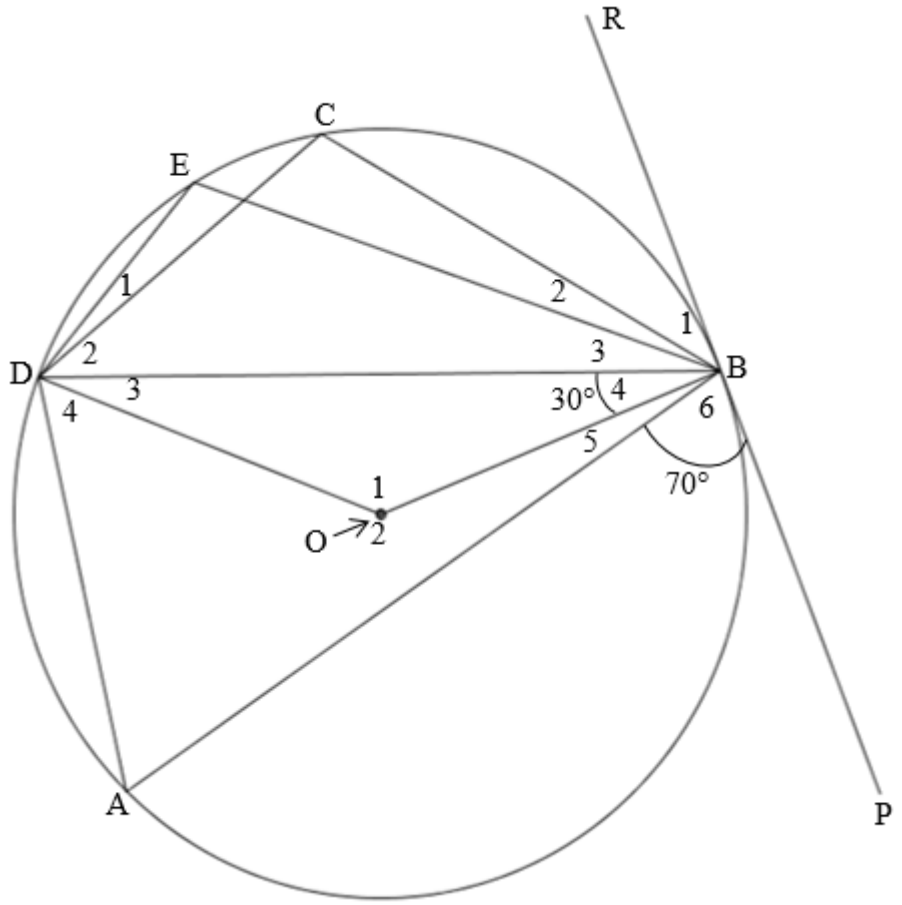
QUESTION 7 / *VRAAG 7*



7.1	$NP^2 = ON^2 + OP^2$ $= (103)^2 + (145)^2$ $\therefore NP = 177,86$	✓ using Pyth theorem correctly *korrekte gebruik van stelling van Pythagoras* ✓ answer / *antwoord*	(2)
7.2	$PQ^2 = (103)^2 + (103)^2 - 2(103)(103).\cos(120°)$ $PQ = 178,40$	✓ subst. into cosine rule *verv. in cosinus formule* ✓ answer / *antwoord*	(2)
7.3	$\cos \hat{N} = \frac{(177,86)^2 + (177,86)^2 - (178.40)^2}{2(177,86)(177,86)}$ $\therefore \hat{N} = 60.20°$	✓ substitution / *vervanging* ✓ $\hat{N}$	(2)
			[6]

QUESTION 8 / *VRAAG 8*			
8.1	Bisects the chord / *Halveer die koord*	✓ answer / *antwoord*	(1)
8.2			
8.2.1	$\hat{Q} = 90°$ [$\angle$ in semi circle.] / *[$\angle$ in halwe sirkel]* $\hat{L}_1 = \hat{Q} = 90°$ [corresp. $\angle^s$, QN ∥ LO] *[ooreenk. $\angle^e$, QN ∥ LO]* QL = LP [line from centre perp. to chord] *[lynstuk vanaf middelpt is loodreg op koord]*	✓ S ✓R ✓ S/R ✓ R	(4)
8.2.2	$MF = 8ML$	✓	(1)
8.2.3	$OP^2 = OL^2 + LP^2$ $(4ML)^2 = (3ML)^2 + 7^2$ $16ML^2 = 9ML^2 + 49$ $7ML^2 = 49$ $\therefore ML = \sqrt{7}$	✓ using Pyth correctly *korrekte gebruik van Pythagoras* ✓ simplification / *vereenvoudiging* ✓ ML	(3)
			[9]

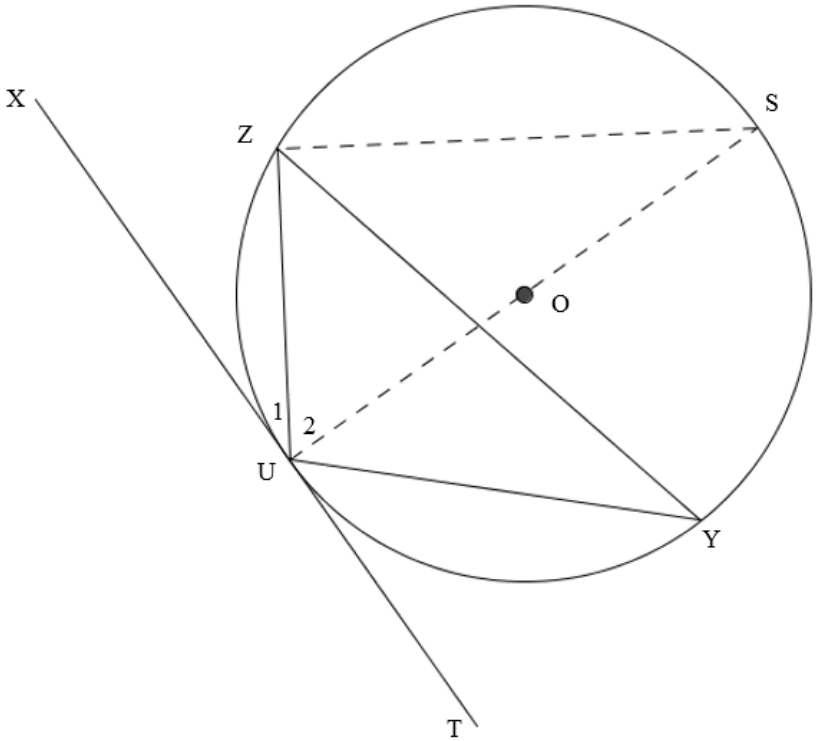
QUESTION 9 / *VRAAG 9*



9.1	$\hat{D}_3 = 30°$ [$\angle$s opp equal sides] /*[$\angle^e$ teenoor gelyke sye]* $\therefore \hat{O}_1 = 120°$ [sum of $\angle$s of a Δ.] / *[som v/d $\angle^e$ van 'n Δ]*	✓ S ✓R ✓ S/R	(3)
9.2	$\hat{A} = 60^0$ [$\angle$ at centre = 2 $\angle$ at circumf.] / *[middelpunts $\angle$ = 2× omtrekshoek]*	✓ S ✓R	(2)
9.3	$\hat{C} = 120°$ [opp. $\angle$s of cyclic quad.] / *[teenoorst. $\angle^e$ van k.v]*	✓ S ✓R	(2)
9.4	$A\hat{D}B = 70°$ [tan chord theorem.] / *[raaklyn koord stelling]*	✓ S ✓R	(2)
			[9]

QUESTION 10 / *VRAAG 10*

10.1



✓ constr.
konstr.

$\hat{U}_2 = 90° - \hat{U}_1$ [tan $\perp$ radius.] / *[raaklyn $\perp$ radius]* ✓ S/R

$\hat{Z} = 90°$ [$\angle$ in semi circle] / *[$\angle$ in halwe sirkel]* ✓ S/R

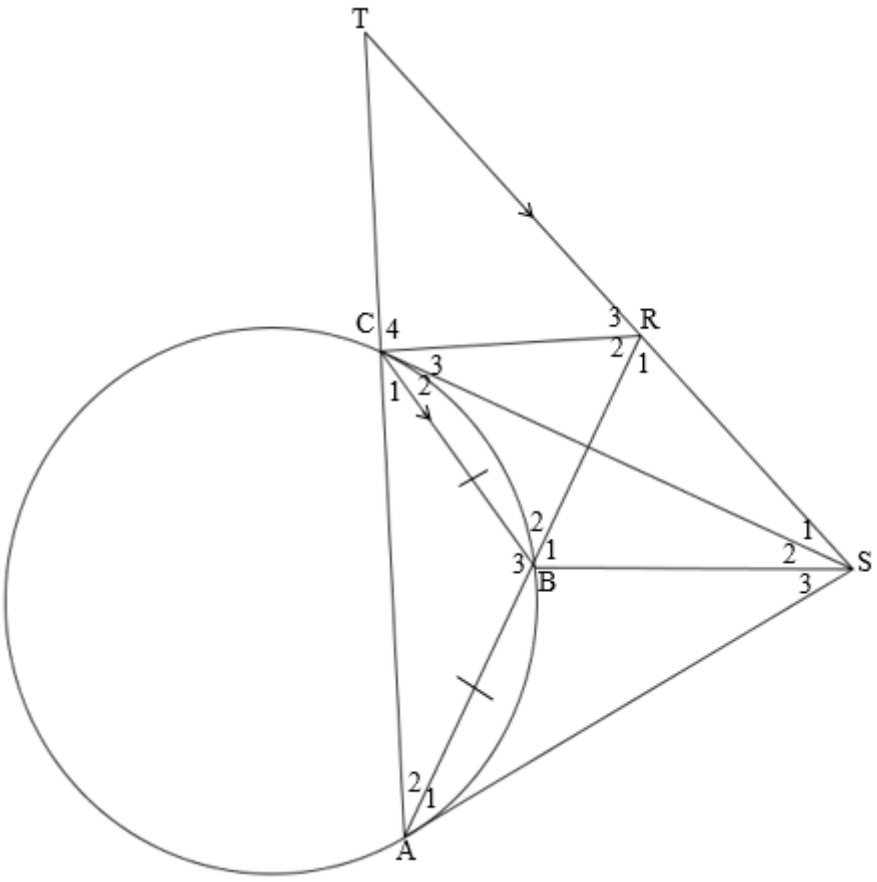
$\hat{S} = 180° - (90° - \hat{U}_1) - 90°$ [sum of $\angle$s of a Δ] / *[som van die $\angle^e$ van 'n Δ]* ✓ S/R

$\therefore \hat{S} = \hat{U}_1$

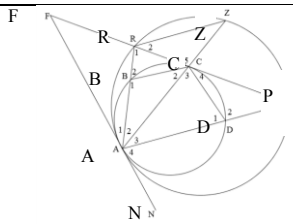
$\hat{S} = \hat{Y}$ [$\angle$s in same segment] / *[$\angle^e$ in dieselfde segment]* ✓ S/R

$\therefore \hat{U}_1 = \hat{Y}$

(5)

10.2			
10.2.1	$\hat{A}_2 = x$ [$\angle$s opp. = sides] / *[$\angle^e$ teenoor gelyke sye]* $\hat{C}_2 = \hat{A}_2 = x$ [tan chord theo.] / *[raaklyn koord stelling]* $\hat{A}_1 = \hat{C}_1$ [tan chord theo.] / *[raaklyn koord stelling]* $\hat{C}_2 = \hat{S}_1$ [alt. $\angle$s , CB $\parallel$ TS] / *[verw. $\angle^e$, CB $\parallel$ TS]* $\hat{C}_1 = \hat{T}$ [corresp $\angle$s , CB $\parallel$ TS] / *[ooreenk. $\angle^e$, CB $\parallel$ TS]*	✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)
10.2.2	$\hat{S}_1 = \hat{T} = x$ [proven in 10.2.1] / *[bewys in 10.2.1]* $\therefore$ CS = CT [sides opp. = $\angle$s] / *[sye teenoor gelyke hoeke]*	✓ S ✓ R	(2)
10.2.3	$\frac{AR}{BR} = \frac{AT}{CT}$ [line $\parallel$ to one side of a Δ] *[lynstuk $\parallel$ aan een sy van 'n Δ]* CS = CT [proved in 10.2.2] / *[bewys in 10.2.2]* $AT = \frac{3}{2} \times 4$ $AT = 6\,cm$	✓ S/R ✓ CS = CT ✓ substitution *vervanging* ✓ AT	(4)
			[16]

QUESTION 11 / *VRAAG 11*



11.1

$\hat{A}_1 = \hat{C}_2$ [tan chord theo.] / *[raaklyn koord stelling]* ✓ S ✓R

$= \hat{Z}$ [tan chord theo] / *[raaklyn koord stelling]* ✓ S/R

$\therefore \hat{C}_2 = \hat{Z}$ [both = *x*] / *[beide = x]*

$\therefore$ BC ‖ RZ [corresp. ∠s =] / *[ooreenk. ∠^{e} =]* ✓ R (4)

11.2

$\hat{Z} = \hat{P}$ [∠s in same segment] / *[∠^{e} in dieselfde segment]* ✓ S/R

$= \hat{C}_2$ [corresp. ∠s ; BC ‖ RZ] / *[ooreenk. ∠^{e} ; BC ‖RZ]* ✓ S

$\therefore$ BC is a tangent to circle ACP [conv. of tan chord theorem]
BC is 'n raaklyn aan sirkel ACP [omgekeerde van raaklyn koord stelling] ✓ R (3)

11.3

$\hat{B}_1 = \hat{D}_2$ [ext. ∠ of a cyclic quad.] ✓ S ✓R
[buite ∠ van koordevierhoek]

$\hat{R} = \hat{B}_1$ [corresp. ∠s , BC‖RZ] ✓ S/R
[ooreenk. ∠^{e} , BC‖RZ]

$\therefore \hat{R} = \hat{D}_2$ ✓ R

$\hat{Z} = \hat{P}$ [∠^{s} in same segment]
[∠^{e} in dieselfde segment]

$\hat{A}_2 = \hat{C}_4$ [3rd ∠] / *[3de ∠]*

$\therefore \Delta$ RZA ||| Δ DPC [equiangular or ∠∠∠]
[gelykhoekig of ∠∠∠] (5)

11.4

$\frac{ZA}{PC} = \frac{RA}{DC}$ [similar Δ^s] / *[gelykvormige Δ^e]* ✓ S/R

$\therefore RA = \frac{ZA \times DC}{PC}$ . . . .(1) ✓ $RA = \frac{ZA \times DC}{PC}$

$\frac{AR}{AB} = \frac{AZ}{AC}$ [line ‖ to one side of a Δ] ✓ S/R
[lynstuk ‖ aan een sy van 'n Δ]

$\therefore AR = \frac{AZ \times AB}{AC}$ . . .(2) ✓ $RA = \frac{AZ \times AB}{AC}$

$\frac{ZA \times DC}{PC} = \frac{AZ \times AB}{AC}$

$\therefore \frac{DC}{PC} \times \frac{AC}{AB} = 1$ ✓ simplification (5)

[17]

TOTAL/TOTAAL: 150