education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE MEMORANDUM

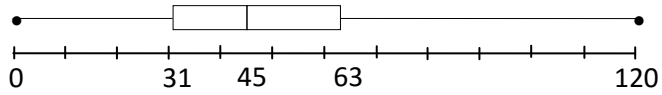
MATHEMATICS MEMORANDUM P2

SEPTEMBER 2016

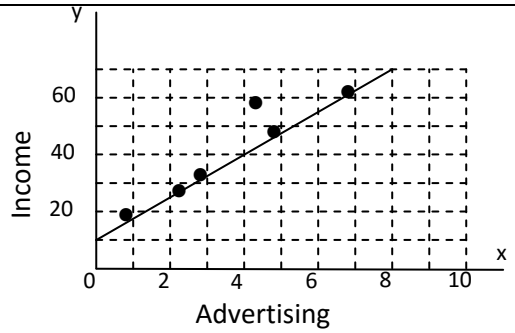
GRADE 12

This memo consists of 15 pages

QUESTION 1

1.1.1	3700	✓ answer (1)
1.1.2	700	✓✓ answer (2)
1.1.3	45 minutes	✓ answer (1)
1.1.4		✓ min / max ✓ $Q_1 + Q_3$ ✓ median (3)
1.2.1	$40 \leq p \leq 54$	✓values ✓ notation (2)
1.2.2	$\frac{32+40+p+56+83}{5}$ $209+p=300$ $p=91$	✓ equation ✓ answer (2)
		[11]

QUESTION 2

2.1		✓2 points plotted correctly ✓4 points plotted correctly ✓6 points plotted correctly (3)
2.2	$y = 7{,}69x + 10{,}49$	✓ 7,69x ✓ 10,49 (2)
2.3	$r = 0{,}95$	✓ answer (1)
2.4	It is a very strong positive correlation.	✓ answer (1)
2.5	$y = 7{,}69(3{,}5) + 10{,}49 = 37{,}405$ Income = R 37 405	✓ substitution ✓ answer (2)
		[9]

QUESTION 3

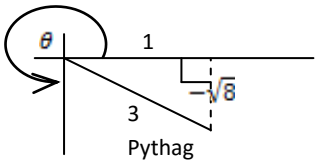
3.1.1	$\frac{x+2}{2}=5$ $\qquad$ $\frac{3+y}{2}=4$ $x+2=10$ $\qquad$ $3+y=8$ $x=8$ $\qquad$ $y=5$ $\therefore B(8;5)$	✓ $x=8$ ✓ $y=5$ (2)
3.1.2	$m_{AM}=\frac{4-3}{5-2}$ $=\frac{1}{3}$ $\perp m=-3$ $y-4=-3(x-5)$ $y-4=-3x+15$ $y=-3x+19$	✓ gradient of AM ✓ perp gradient ✓ correct sub into equation ✓ answer (4)
3.1.3	$0=-3x+19$ $x=\frac{19}{3}$ $\text{C}\left(\frac{19}{3};0\right)$ $m_{AC}=\frac{3-0}{2-\frac{19}{3}}$ $=\frac{-9}{13}$ $\tan A\hat{C}X=\frac{-9}{13}$ $A\hat{C}X=180°-34{,}7°$ $=145{,}3°$ $\alpha=34{,}7°$	✓ y = 0 ✓ coordinates of C ✓ gradient of AC ✓ 145,3º ✓ answer (5)

3.1.4	BC is a diameter converse of $\angle$ in a semi circle	✓S ✓R (2)
3.2.1	$D\left(\frac{-4+8}{2};\frac{10+2}{2}\right)$ $D(2;6)$ $AD=\sqrt{(7-2)^2+(11-6)^2}$ $=\sqrt{25+25}$ $=\sqrt{50}$ $=5\sqrt{2}$	✓✓ coordinates of D ✓ sub in distance formula ✓ answer (4)
3.2.2	$AD=\frac{1}{2}DE$ $D(2;6)$ $E(x;-4)$ $5\sqrt{2}=\frac{1}{2}\sqrt{(x-2)^2+(-4-6)^2}$ $10\sqrt{2}=\sqrt{x^2-4x+4+100}$ $\left(10\sqrt{2}\right)^2=x^2-4x+104$ $0=x^2-4x-96$ $0=(x-12)(x+8)$ $x=12$ or $x=-8$ NA	✓ sub in distance formula ✓ simplification ✓ standard form ✓ factors ✓ $x=-8$ only (5)
3.3	$m_{AB}=m_{AC}$ $\frac{y-4}{-3+7}=\frac{2-4}{5+7}$ $12(y-4)=-8$ $12y=40$ $y=\frac{10}{3}$	✓ equating gradients ✓ simplification ✓answer (3)
		[25]

QUESTION 4

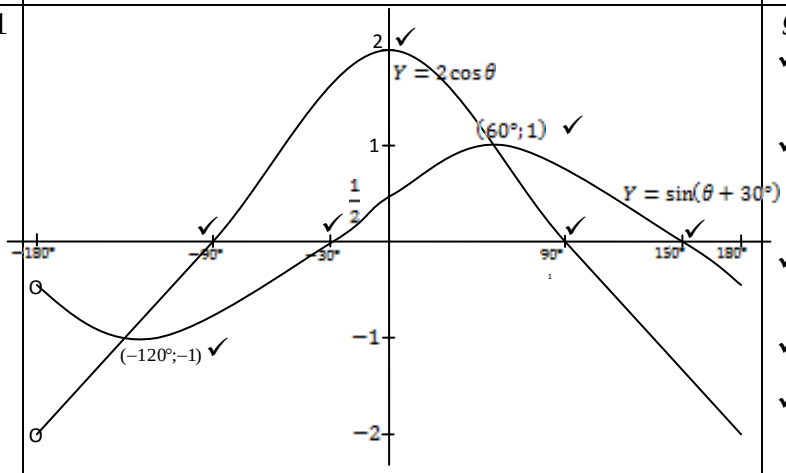
4.1	$x^2+2x+1+y^2-4y+4=44+1+4$ $(x+1)^2+(y-2)^2=49$ $\therefore M(-1;2)$	✓✓equation of circle ✓ answer (3)
4.2	Line $AK: y=-x+5$ $let A(a;b) then: A(a;-a+5)$ $m_{AM}=1$ $\frac{-a+5-2}{a-(-1)}=1$ $-a+3=a+1$ $a=1$ $b=4$ $\therefore A(1;4)$	✓standard form ✓coordinates of A ✓substitution (3)
4.3	$r^2=AM^2=(1+1)^2+(4-2)^2=8$ $(x+1)^2+(y-2)^2=8$	✓sub and ✓r value ✓ equation (3)
4.4	Distance = radius of bigger circle – radius of smaller circle = 7 - $\sqrt{8}$ = 7 - $2\sqrt{2}$ or 4,17	✓ method ✓ answer (2)
4.5	$K(5;0)$ $AK=\sqrt{(5-1)^2+(4-0)^2}$ $=\sqrt{32}$ Area $\Delta AMK=\frac{1}{2}AM.AK$ $=\frac{1}{2}\sqrt{8}\sqrt{32}$ $=8$	✓ $x=5$ ✓ $y=0$ ✓length of AK ✓sub in area formula ✓answer (5)
		[16]

QUESTION 5

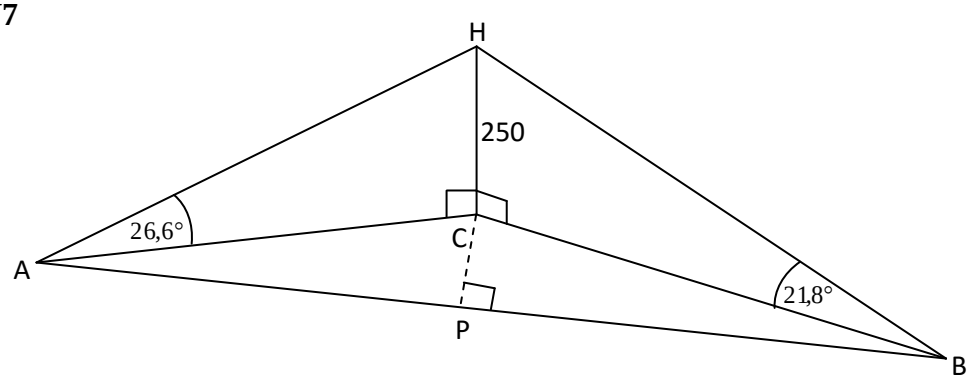
5.1	$\sqrt{8}\sin\theta+\cos\theta$ $=\left(\sqrt{8}\right)\left(-\frac{\sqrt{8}}{3}\right)+.\frac{1}{3}$  $=-\frac{7}{3}$	✓ correct sketch in correct quadrant ✓ $r=3$ ✓ $\frac{1}{3}$ ✓ $-\frac{\sqrt{8}}{3}$ ✓ answer (5)
5.2	$\frac{(-\cos 45°)^2.(\tan x).(-\sin x)}{-\sin x}$ $=\left(-\frac{1}{\sqrt{2}}\right)^2.\tan x$ $=\frac{1}{2}\tan x$	✓ ✓✓✓ reduction formulae ✓ value of special angle ✓ answer (6)
		[11]

QUESTION 6

6.1.1	$2\cos\theta=\sin(\theta+30°)$ $=\sin\theta\cos30°+\cos\theta\sin30°$ $=\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta$ $\therefore 4\cos\theta=\sqrt{3}\sin\theta+\cos\theta$ $\therefore 3\cos\theta=\sqrt{3}\sin\theta$	✓ expansion ✓ substitution of special angles ✓ $4\cos\theta=\sqrt{3}\sin\theta+\cos\theta$ (3)

6.1.2	$3\cos\theta = \sqrt{3}\sin\theta$ $\frac{\sin\theta}{\cos\theta} = \frac{3}{\sqrt{3}}$ $\tan\theta = \frac{3}{\sqrt{3}}$ $\theta = 60° + 180.n$ $\theta = -120° \text{ or } \theta = 60°$	✓ division ✓ $\tan\theta = \frac{3}{\sqrt{3}}$ ✓✓ answers (4)
6.2.1		g : ✓ y-intercept and x- intercepts ✓ shape f : ✓ y-intercept and x-intercepts ✓ turning points ✓ shape (5)
6.2.2	$-30° < \theta < 90°$	✓✓ values ✓ notation (3)
6.2.3	$-120° < \theta < 60°$	✓✓ values ✓ notation (3)
		[18]

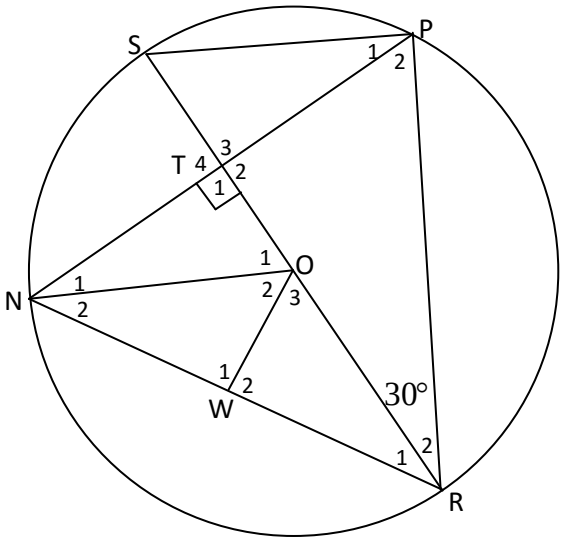
QUESTION7



7.1	*In* ΔAHC; $A\hat{H}C = 63{,}4°$ $\frac{AC}{\sin 63{,}4°} = \frac{250}{\sin 26{,}6°}$ $AC = 624{,}43$ *In* $\Delta CHB : C\hat{H}B = 68{,}2°$ $\frac{BC}{\sin 68{,}2°} = \frac{250}{\sin 21{,}8°}$ $BC = 625{,}04$ *In* ΔABC $AB^2 = (624{,}43)^2 + (625{,}04)^2 - 2(624{,}43)(625{,}04)\cos 104{,}5°$ $= 976031{,}3165$ $AB = 987{,}94$	✓ method ✓ value of AC ✓ method ✓ value of BC ✓ sub in cos-rule ✓ answer (6)
7.2	*In* $\Delta ABC : \frac{\sin A}{625{,}04} = \frac{\sin 104{,}5°}{987{,}94}$ $\sin A = 0{,}6317$ $\hat{A} = 39{,}2°$	✓ method ✓ $\sin A = 0{,}6317$ ✓ answer (3)
		[9]

QUESTION 8

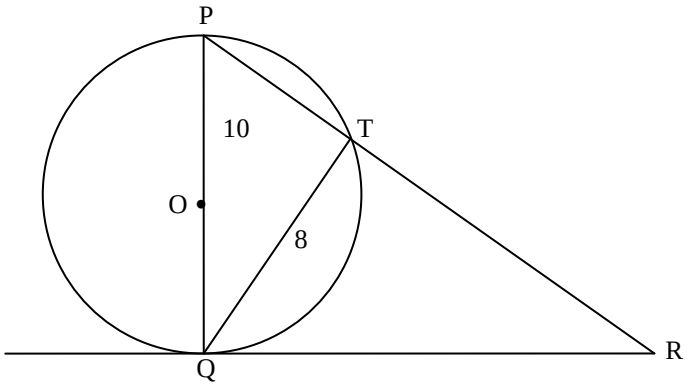
8.1



8.1.1	$\hat{P}_1 + \hat{P}_2 = 90°$ $\hat{S} + \hat{P}_1 + \hat{P}_2 + \hat{R}_2 = 180°$ $\hat{S} + 90° + 30° = 180°$ $\therefore\ \hat{S} = 60°$	∠ in semi circle $\angle's\ in\ \Delta$	✓ S✓ R ✓ answer (3)
8.1.2	$\hat{S} = \hat{N}_1 + \hat{N}_2$ $\hat{N}_1 + \hat{N}_2 = 60°$ $\hat{N}_1 + \hat{N}_2 + \hat{T}_1 + \hat{R}_1 = 180°$ $60° + 90° + \hat{R}_1 = 180°$ $\therefore\ \hat{R}_1 = 30°$ OR $\hat{T}_4 + \hat{P}_1 + \hat{S} = 180°$ $90° + \hat{P}_1 + 60° = 180°$ $\hat{P}_1 = 30°$ $\hat{P}_1 = \hat{R}_1$ $\therefore \hat{R}_1 = 30°$	PR subt = $\angle's$ $\angle's\ in\ \Delta$ $\angle's\ in\ \Delta$ NR subtends = $\angle's$	✓ S ✓ R ✓ answer (3) ✓ $\hat{P}_1 = 30°$ ✓ S/ R ✓ answer (3)

8.1.3	$\hat{O}_1 = 2\hat{R}_1$ ∠ at centre = 2.∠ on circumference $\therefore \hat{O}_1 = 2(30°) = 60°$ $\hat{N}_1 + \hat{T}_1 + \hat{O}_1 = 180°$ ∠'s in Δ $\hat{N}_1 + 90° + 60° = 180°$ $\hat{N}_1 = 30°$ OR $\hat{O}_1 = 2(30°) = 60°$ ∠ at center = 2 . ∠ at circumference $N\hat{O}_1R = 120°$ adj. on str. line $\hat{N}_2 = \hat{R}_1$ ∠'s opp = sides $N\hat{O}_1R + 2\hat{N}_2 = 180°$ ∠'s in Δ $120° + 2\hat{N}_2 = 180°$ $2\hat{N}_2 = 180° - 120°$ $\hat{N}_2 = 30°$ But $\hat{N}_1 + \hat{N}_2 = 60°$ $\therefore \hat{N}_1 = 30°$	✓ S ✓ R ✓ answer (3) ✓S/R ✓ S/R ✓ answer (3)
8.1.4	$NW = WR$ given $\therefore \hat{W}_1 = 90°$ line from centre, midpoint chord $\hat{T}_1 = 90°$ $\therefore \hat{W}_1 + \hat{T}_1 = 180°$ $\therefore TNWO$ is a cyclic quad opp ∠'s suppl	✓ S ✓ S/R ✓ S ✓R (4)

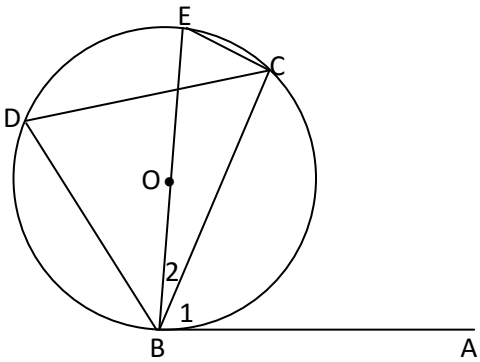
8.2



8.2.1	$P\hat{T}Q = 90°$ ∠ in semi-circle $PQ^2 = QT^2 + PT^2$ Pythagoras theorem $10^2 = 8^2 + PT^2$ PT = 6	✓ S ✓R ✓ S (3)
8.2.2	$P\hat{T}Q = 90°$ rad ⊥ tangent $PR^2 = PQ^2 + QR^2$ Pythagoras theorem $(x+6)^2 = 10^2 + QR^2$ $x^2 + 12x - 64 = QR^2$ $\sqrt{x^2 + 12x - 64} = QR$	✓ S/R ✓substitution (2)
8.2.3	$QR^2 = QT^2 + TR^2$ Pythagoras theorem $QR^2 = 8^2 + x^2$ $(x+6)^2 - 10^2 = 8^2 + x^2$ $x^2 + 12x - 64 = 64 + x^2$ $12x = 128$ $x = \frac{32}{3}$	✓ S ✓ substituting QR^2 ✓ answer (3)
		[21]

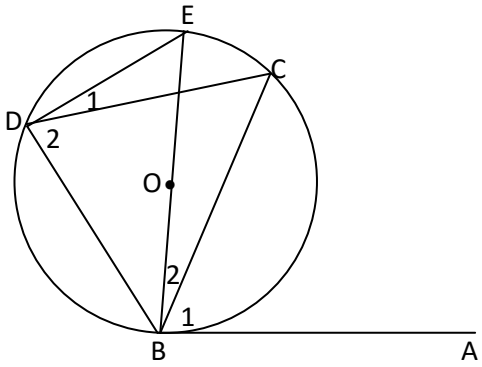
QUESTION 9

9.1



Draw diameter OBE and EC	constr.	✓ construction
$\hat{B}_1 + \hat{B}_2 = 90°$	radius $\perp$ tangent	✓ S/R
$\hat{C} = 90°$	$\angle$ in semi circle	
$\hat{E} + \hat{B}_2 = 90°$	$\angle$s in Δ	✓ S/R
$\hat{E} + \hat{B}_2 = 90°$		
$\therefore \hat{B}_1 = \hat{E}$		✓ S
but $\hat{D} = \hat{E}$	subt BD	✓ S/R
$\therefore \hat{B}_1 = \hat{D}$		(5)

OR



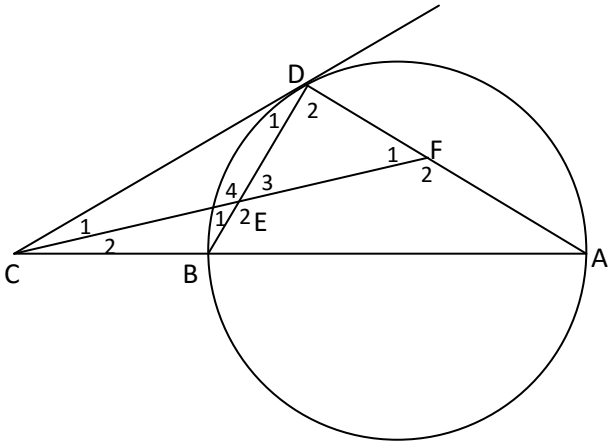
Draw diameter BOE and ED	constr.	✓ construction
$\hat{B}_1 + \hat{B}_2 = 90°$	radius $\perp$ tangent	✓ S/R
$\hat{D}_1 + \hat{D}_2 = 90°$	$\angle$ in semi circle	✓ S/R
But $\hat{D}_1 = \hat{B}_2$	EC subtends = $\angle$s	✓ S/R
But $\hat{D} = \hat{E}$		
$\therefore \hat{B}_2 = \hat{D}_2$ or $A\hat{B}C = \hat{D}$		✓ S (5)

9.2

$AB = 6$

$CD = 4$

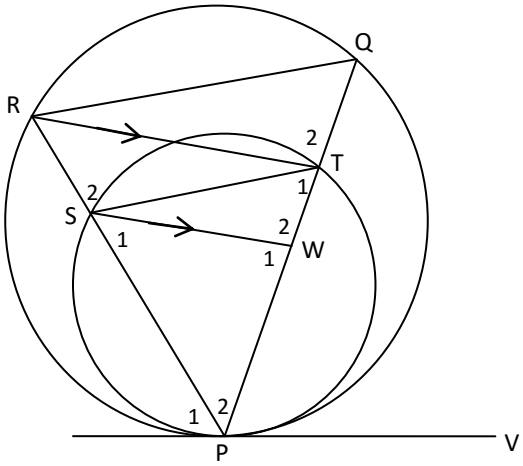
$\hat{D}_2 = 90°$



9.2.1	In ΔDBC *and* ΔADC , $\hat{D}_1 = \hat{A}$ tan-chord $D\hat{C}A = D\hat{C}A$ common $\therefore\ \Delta DBC \,///\, \Delta ADC$ $\angle, \angle, \angle$	✓ S/R ✓ S ✓ R (3)
9.2.2	$\frac{DC}{BC} = \frac{AC}{DC}$ $\Delta DBC \,///\, \Delta ADC$ $DC^2 = BC.AC$ $4^2 = BC(AB + BC)$ $= BC(6 + BC)$ $16 = 6BC + BC^2$ $BC^2 + 6BC - 16 = 0$ $(BC + 8)(BC - 2) = 0$ $BC = -8 \text{ or } BC = 2$ $\therefore BC = 2$	✓ correct ratio ✓ simplification ✓ substitution ✓ std form ✓ factors ✓ $BC = 2$ only (6)
9.2.3	$\Delta DEC \,///\, \Delta AFC$ given $\frac{DC}{EC} = \frac{AC}{FC}$ $\frac{4}{EC} = \frac{8}{FC}$ $\therefore EC = \frac{1}{2}FC$ $\therefore EC = EF$	✓ S ✓ correct ratio ✓ AC = 8 ✓ simplification (4)

9.2.4	$\frac{\Delta CDF}{\Delta CFA} = \frac{1}{2}$ $FA = 2FD$ given $2\Delta CDF = \Delta CFA$ but $\Delta CDF + \Delta CFA = \Delta ADC$ $\therefore\ \Delta CDF + 2\Delta CDF = \Delta ADC$ $\therefore\ 3\Delta CDF = \Delta ADC$ and $\Delta CDE = \frac{1}{2}\Delta CDF$ $\frac{\Delta CDE}{\Delta ACD} = \frac{\frac{1}{2}\Delta CDF}{3\Delta CDF}$ $= \frac{1}{6}$	✓S ✓R ✓ S ✓ S ✓ answer (5)
		[23]

QUESTION 10



10.1	$\hat{P}_1 = \hat{T}_1$ tan chord theorem $\hat{P}_1 = \hat{Q}$ tan chord theorem $\hat{T}_1 = \hat{Q}$ $\therefore$ RS // ST corresp $\angle s$ are equal	✓ S ✓R ✓ S ✓ R (4)
10.2	$\frac{PS}{SR} = \frac{PW}{WT}$ lines // one side of Δ $\frac{PS}{SR} = \frac{PT}{TQ}$ $\therefore \frac{PW}{WT} = \frac{PT}{TQ}$	✓ S ✓ R ✓ S (3)
		[7]

TOTAL: 150