education
DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P2

SEPTEMBER 2016

GRADE 12

MARKS: 150

TIME: 3 Hours

This question paper consists of 13 pages including the formula sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Answer all the questions in the ANSWER BOOK provided.
4. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
5. Answers only will NOT necessarily be awarded full marks.

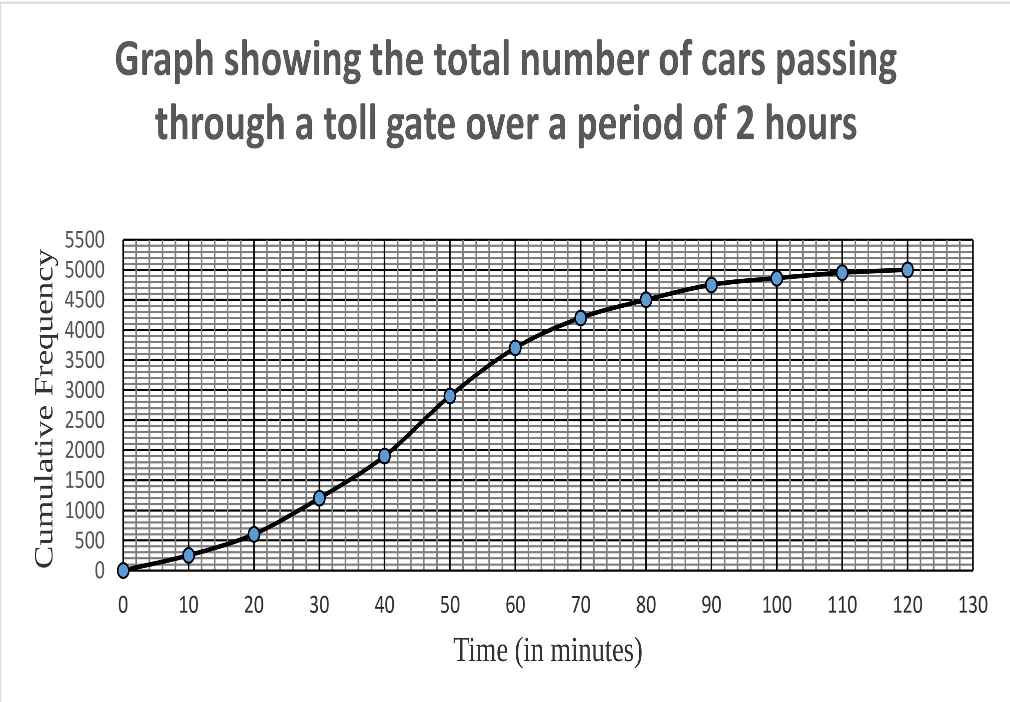
6 You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1.1 The data in the table and the ogive below represents the number of cars passing through a toll gate over a 2 hour peak period during a long weekend. The counting of cars started at 13:00.

Time in minutes	0	10	20	30	40	50	60	70	80	90	100	110	120
Cumulative frequency	0	250	600	1200	1900	2900	3700	4200	4500	4750	4860	4950	5000



1.1.1 How many cars passed through the toll gate in the first hour? (1)

1.1.2 How many cars passed through the toll gate between 13:30 and 13:40? (2)

1.1.3 After how long had 2 500 cars passed through the toll gate? (1)

1.1.4 Draw a box and whisker diagram for the above data. (3)

1.2 The following data values are given in the table below:

40	83	32	54	*p*

1.2.1 What is the value of p if the mean for the data set is 60? (2)

1.2.2 Determine the possible values of p if p is the median of the data set. (2)

[11]

QUESTION 2

The table below shows the amount of money spent on advertising and the corresponding income of the company, in thousands of rand, per month over a 6-month period.

Month	1	2	3	4	5	6
Advertising	3	4,5	1	5	7	2,4
Income	32	56	18	48	60	25

2.1 Draw a scatter plot for the data. (3)

2.2 Determine the equation of the least squares regression line. (2)

2.3 Determine the correlation coefficient of the data. (1)

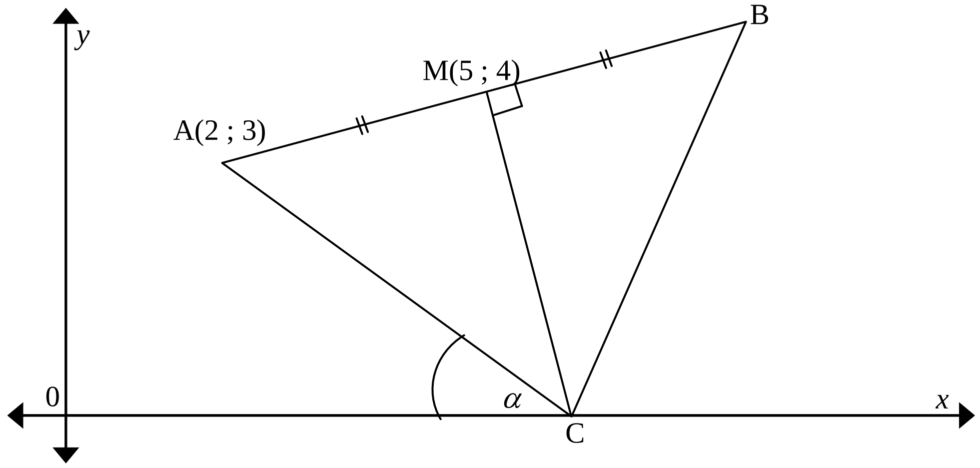
2.4 Comment on the strength of the correlation coefficient in QUESTION 2.3. (1)

2.5 Predict the company's income in a month where R3 500 is spent on advertising. (2)

[9]

QUESTION 3

3.1 In the following sketch AMB is a straight line with AM = MB and CM $\perp$ AB.



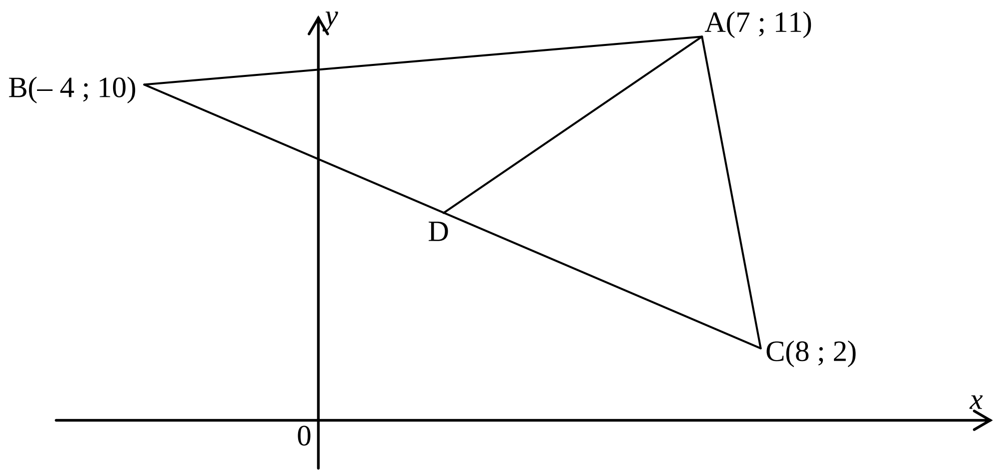
3.1.1 Calculate the coordinates of B. (2)

3.1.2 Determine the equation of MC. (4)

3.1.3 Calculate the size of α , correct to one decimal place. (5)

3.1.4 State with a reason why the vertices of triangle BCM lie on the circumference of a circle. (2)

3.2 In the figure below, A(7 ; 11), B(– 4 ; 10) and C(8 ; 2) are the vertices of a triangle and D is the midpoint of BC.



3.2.1 Calculate the length of AD and leave the answer in its simplest surd form. (4)

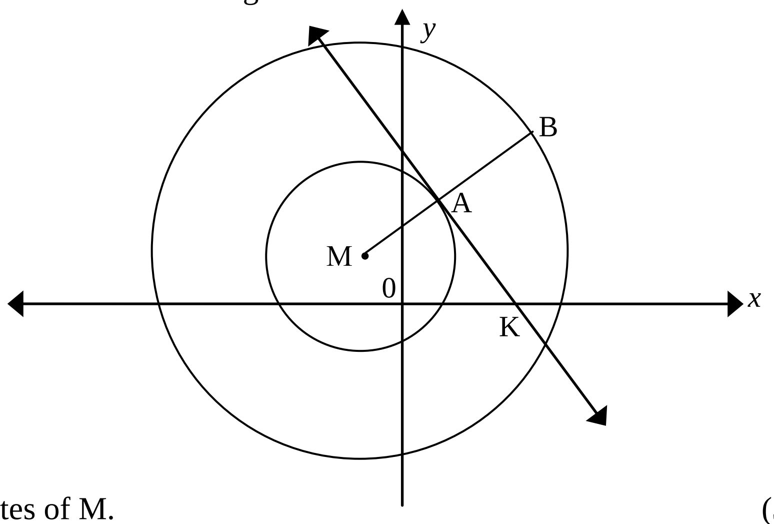
3.2.2 If AD is produced to the point E(x ; – 4) in the fourth quadrant such that $AD = \frac{1}{2}DE$ calculate the value of x. (5)

3.3 If A(–7 ; 4), B(–3 ; y) and C(5 ; 2) are collinear, calculate the value of y. (3)

[25]

QUESTION 4

In the figure, M is the centre of two concentric circles (i.e. the circles have a common centre). The larger circle has equation $x^2 + y^2 = 4y - 2x + 44$. The line $y + x - 5 = 0$ is the tangent to the smaller circle at A. K is the x-intercept of the tangent AK. B is a point on the larger circle such that MAB is a straight line.



4.1 Determine the coordinates of M. (3)

4.2 Show that the coordinates of A are A(1 ; 4). (3)

4.3 Determine the equation of the smaller circle. (3)

4.4 Calculate the length of AB. (2)

4.5 The straight line $y + x - 5 = 0$ meets the straight line $y = 0$ at point K.

Calculate the area of Δ AMK. (5)

[16]

QUESTION 5

This question must be answered without the use of a calculator.

5.1 If $\tan\theta = -\sqrt{8}$ and $\theta \in [180°;360°]$ determine by using a sketch, the value of $\sqrt{8}\sin\theta + \cos\theta$. (5)

5.2 Simplify to a single trigonometric ratio of x:

$$\frac{\cos^2 225.\tan(180° + x).\cos(90° + x)}{\sin(-x)}$$ (6)

[11]

QUESTION 6

6.1 Given the equation $2\cos\theta = \sin(\theta + 30°)$.

6.1.1 Show that $2\cos\theta = \sin(\theta + 30°)$ is equivalent

to $\sqrt{3}\sin\theta = 3\cos\theta$. (3)

6.1.2 Hence or otherwise, calculate θ if $\theta \in [-180°;180°]$. (4)

6.2 Consider the functions $f(\theta) = 2\cos\theta$ and $g(\theta) = \sin(\theta + 30°)$ for $\theta \in [-180°;180°]$

6.2.1 Sketch graphs of f and g on the same set of axes. Show intercepts on the axes clearly. (5)

6.2.2 Use the graph to determine $\theta \in [-90°;90°]$ if:

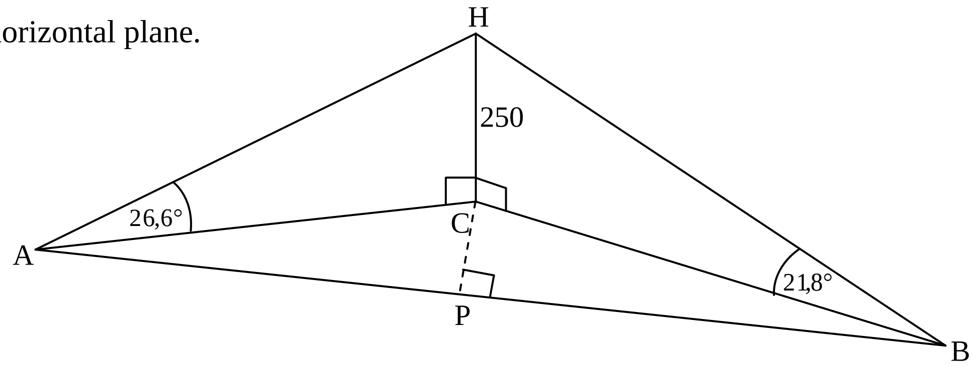
$f(\theta).g(\theta) > 0$ (3)

6.2.3 For which values of θ will $g'(\theta) > 0$, $\theta \in [-180°;180°]$? (3)

[18]

QUESTION 7

Two observers at A and B sight a helicopter H hovering at 250 *m* directly above C. The angles of elevation from A and B to H respectively are $26,6°$ and $21,8°$. A, B and C are in the same horizontal plane.



7.1 Calculate the distance between the observers A and B if $A\hat{C}B = 104,5°$. (6)

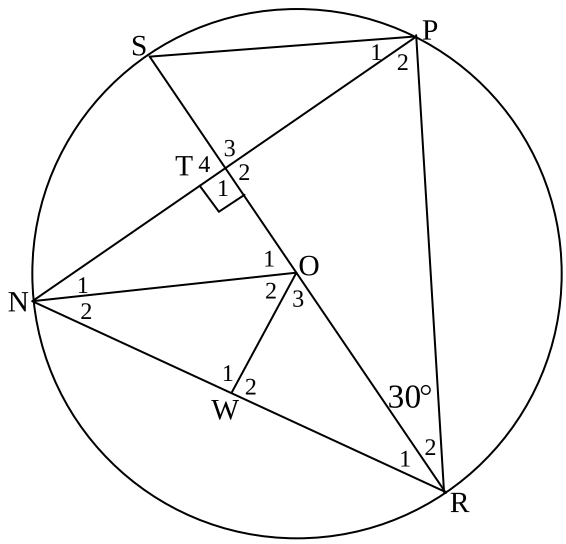
7.2 Calculate $C\hat{A}B$. (3)

[9]

Provide Euclidean geometry reasons for all statements in Question 8 to Question 10

QUESTION 8

8.1 In the diagram, the vertices of ΔPNR lie on the circle with centre O. Diameter SR and chord NP intersect at T. Point W lies on NR. OT $\perp$ NP. $\hat{R}_2 = 30°$.



Determine, stating reasons, the size of:

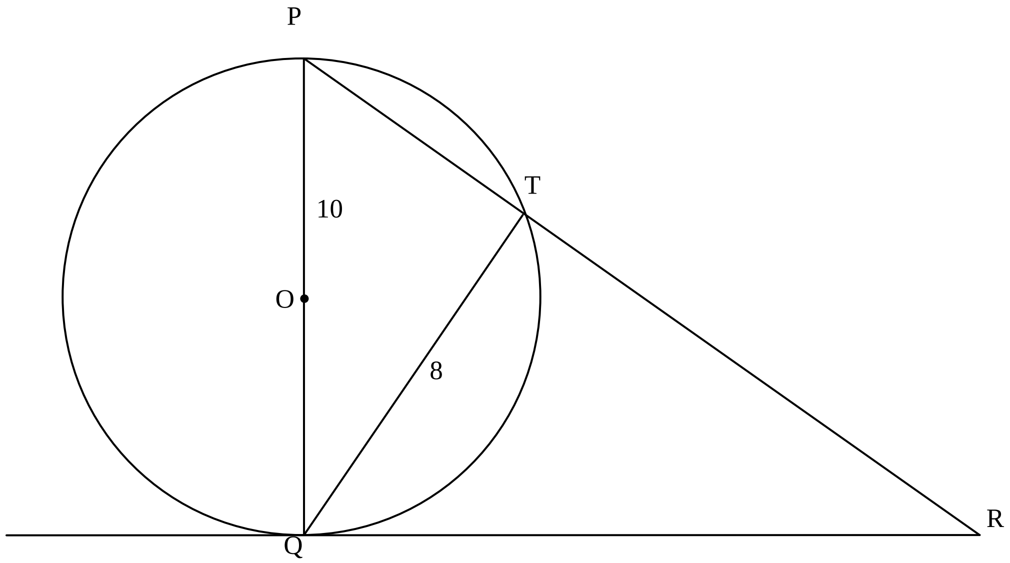
8.1.1 $\hat{S}$ (3)

8.1.2 $\hat{R}_1$ (3)

8.1.3 $\hat{N}_1$ (3)

8.1.4 If it is further given that NW = WR, prove that TNWO is a cyclic quadrilateral. (4)

8.2 In the figure below, PQ is a diameter of the circle with centre O. PT is a chord of a circle which is produced to R. PQ = 10, QT = 8 and TR = x



8.2.1 Show that PT = 6 (3)

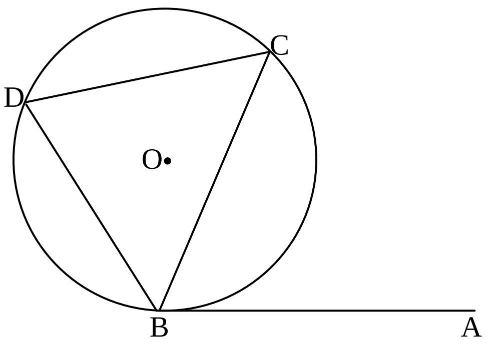
8.2.2 Show that QR = $\sqrt{x^2 + 12x - 64}$ in ΔPQR. (2)

8.2.3 Calculate the value of x. (3)

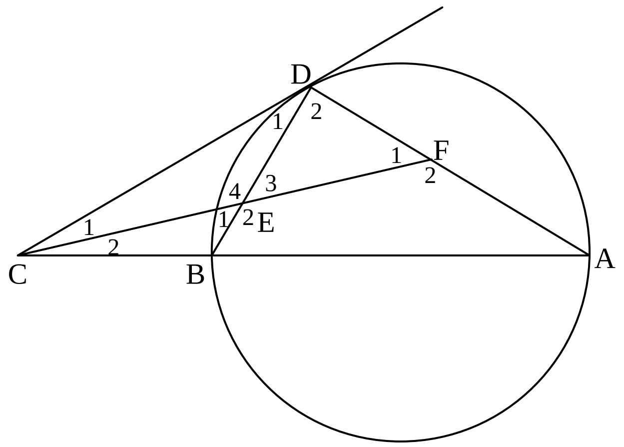
[21]

QUESTION 9

9.1 In the accompanying figure, AB is a tangent to the circle at B. O is the centre of the circle. Draw the diagram and prove the theorem which states that if AB is a tangent to the circle at B, then $A\hat{B}C = \hat{D}$. (5)



9.2 In the figure, AB is a diameter of the circle The tangent to the circle at D meets AB produced at C. The bisector of $\hat{C}$ cuts DB at E and meets AD at F. The radius of the circle is 3 *units* and the length of CD is 4 *units*.



9.2.1 Prove that $\Delta DBC \parallel\!| \Delta ADC$. (3)

9.2.2 Calculate the length of BC. (6)

9.2.3 If $\Delta DEC \parallel\!| \Delta AFC$, show that CE = EF. (4)

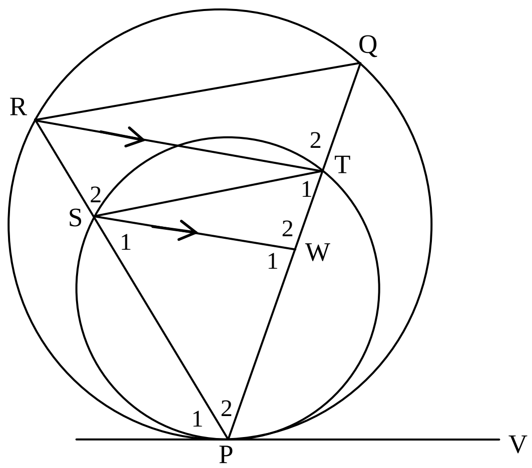
9.2.4 If $\frac{DF}{FA} = \frac{1}{2}$ calculate the numerical value of the ratio:

$$\frac{\text{area of } \Delta CDE}{\text{area of } \Delta ACD}$$ (5)

[23]

QUESTION 10

Circles STP and RQP touch internally at P. VP is a common tangent to the circles at P. W is a point on PQ such that SW || RT. Chords RQ, ST, PQ and PR are drawn.



10.1 Prove that RQ || ST (4)

10.2 Prove that $\frac{PW}{WT} = \frac{PT}{TQ}$ (3)

[7]

TOTAL: 150

FORMULA SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + in)$ $\quad A = P(1 - in)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 \pm i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad T_n = ar^{n-1}$

$S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}\ ;\ -1 < r < 1$ $\quad F = \frac{x[(1+i)^n - 1]}{i}$

$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $\quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan \vartheta$

$(x - a)^2 + (y - b)^2 = r^2$ $\quad$ In $\Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

Area of $\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha \sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha \sin\beta$

$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$ $\quad \cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2\sin^2 A \\ 2\cos^2 A - 1 \end{cases}$$

$\sin 2A = 2\sin A.\cos A$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\bar{x} = \frac{\sum x}{n}$ $\quad y = a + bx$

$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})}$ $\quad \sigma^2 = \frac{\sum_{t=1}^{n} (x_1 - \bar{x})^2}{n}$