Western Cape Government
Education

YOUR SCHOOL NAME

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATH.2

MATHEMATICS P2

SEPTEMBER 2016

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

Turn over please

QUESTION 1

Learners were asked the time (to the nearest minute) that they usually take to get from home to school each morning. The results are shown in the table below:

Time in minutes	**Number of learners**
$5 < t \leq 10$	160
$10 < t \leq 15$	150
$15 < t \leq 20$	110
$20 < t \leq 25$	60
$25 < t \leq 30$	45
$30 < t \leq 35$	15

1.1 Complete the cumulative frequency column in the ANSWER BOOK provided. (2)

1.2 Draw the ogive of these data on the grid provided in the ANSWER BOOK. (4)

1.3 Calculate the mean time that these learners take to get to school in the morning. (2)

1.4 Calculate the standard deviation of this data. (2)

1.5 Use the ogive to determine approximately how many learners would take more than one standard deviation from the average time it takes learners to get to school. (3)

[13]

QUESTION 2

The table shows the number of athletes and total amount of medals that some countries obtained during the 2012 Summer Olympics.

(Information according to: http://www.wow.com/wiki/2012_London_Olympics)

Country	**Number of athletes**	**Number of medals**
USA	530	103
China	396	88
Great Brittan	541	65
Russia	436	79
South Korea	245	28
Germany	392	44
France	330	34
Australia	410	35
South-Africa	133	6
Canada	277	18
Netherland	175	20
New Zealand	184	13

2.1 Use the table to determine the equation of the least squares regression line for the data. (3)

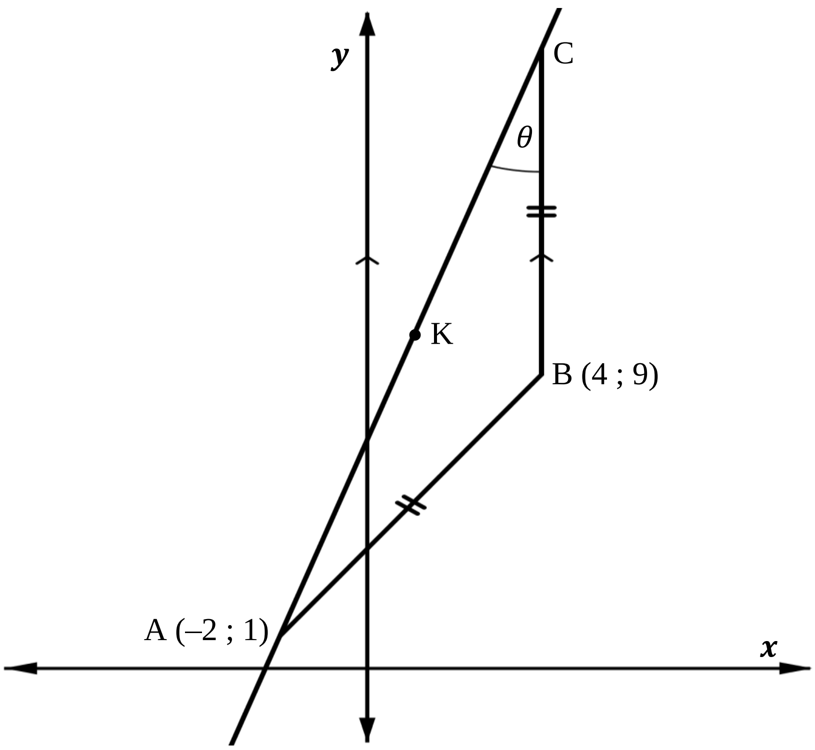
2.2 Calculate the correlation coefficient for the data. (1)

2.3 How many medals (to the nearest integer) can be predicted, will a country with 350 athletes obtain? (2)

2.4 Comment on the correlation between the number of participating athletes and the number of medals obtained by a country. (1)
[7]

QUESTION 3

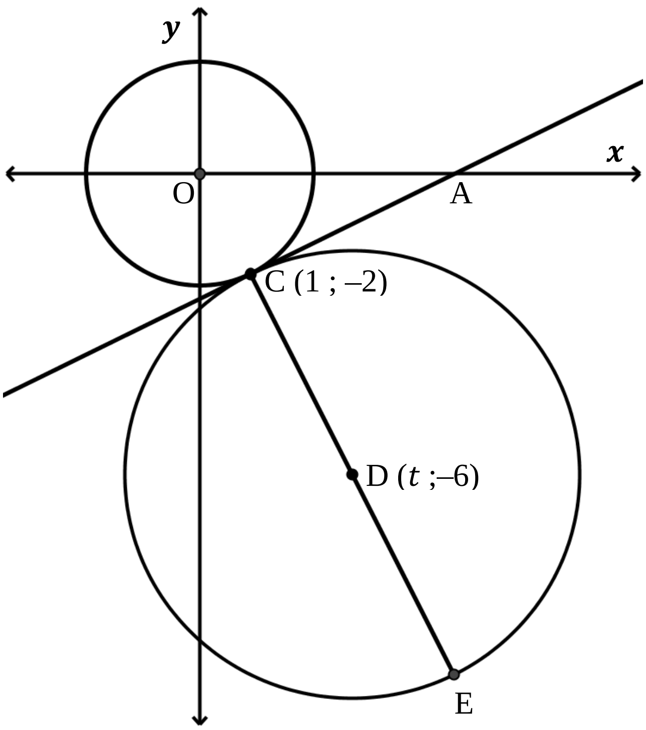
In the diagram below, ABC is an isosceles triangle with A(–2;1) and B(4 ; 9). AB = BC and BC is parallel to the y-axis.



3.1	Calculate the length of AB.	(2)
3.2	Calculate the coordinates of C.	(2)
3.3	Calculate the coordinates of K, the midpoint of AC.	(2)
3.4	Determine the equation of AC in the form $y = mx + c$.	(3)
3.5	Calculate the size of θ.	(4)
3.6	Calculate the area of ΔABC.	(4)
		[17]

QUESTION 4

The diagram below consists of two circles, which touch each other externally at C (1 ; –2). The smaller circle has its centre O at the origin. The other circle has centre D(t ; –6). CA is a common tangent which intersects the x-axis at A. CDE is the diameter of the larger circle.



4.1 Give a reason why the points O, C and D lie on a straight line. (2)

4.2 Calculate the gradient of OC. (2)

4.3 Hence, show that the value of $t = 3$. (2)

4.4 Determine the equation of the tangent AC in the form $y = mx + c$. (3)

4.5 Calculate the coordinates of E. (2)

4.6 Determine the equation of a circle passing through the points A(5 ; 0), C and E in the form $(x-a)^2+(y-b)^2=r^2$. (6)

4.7 If a circle centre D with equation $(x-3)^2+(y+6)^2=r^2$ has to cut the circle centre O twice, give all possible values of r. (4)

[21]

QUESTION 5

5.1 If $\sin 34^\circ = p$, determine the value of each of the following in terms of p, **WITHOUT USING A CALCULATOR.**

5.1.1 $\sin 214^\circ$ (2)

5.1.2 $\cos 34^\circ . \cos(-22^\circ) + \cos 56^\circ . \sin 338^\circ$ (4)

5.1.3 $\cos 68^\circ$ (2)

5.2 Determine the value of each of the following expressions:

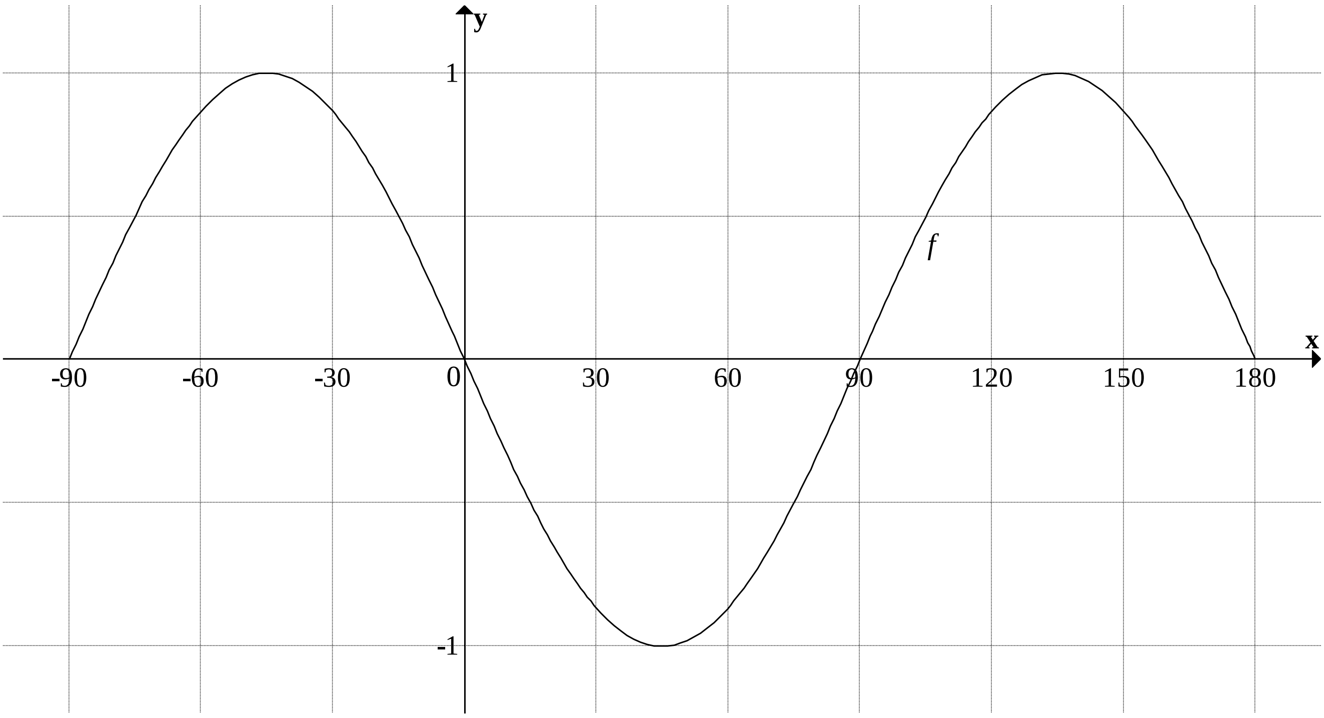
5.2.1 $\dfrac{\cos(90^\circ - 2\theta).\sin\theta}{\sin^2(180^\circ + \theta).\cos(720^\circ + \theta)}$ (6)

5.2.2 $\dfrac{1}{\sin^2 2x} - \dfrac{1}{\tan^2 2x}$ (4)

[18]

QUESTION 6

In the diagram, the graph of $f(x) = -\sin 2x$ is drawn for the interval $x \epsilon [-90°; 180°]$.



6.1 Draw the graph of g, where $g(x) = \cos(x - 60°)$, on the same system of axes for the interval $x \epsilon [-90°; 180°]$ in the ANSWER BOOK. (3)

6.2 Determine the general solution of $f(x) = g(x)$. (5)

6.3 Use your graphs to solve x if $f(x) \leq g(x)$ for $x \epsilon [-90°; 180°]$ (3)

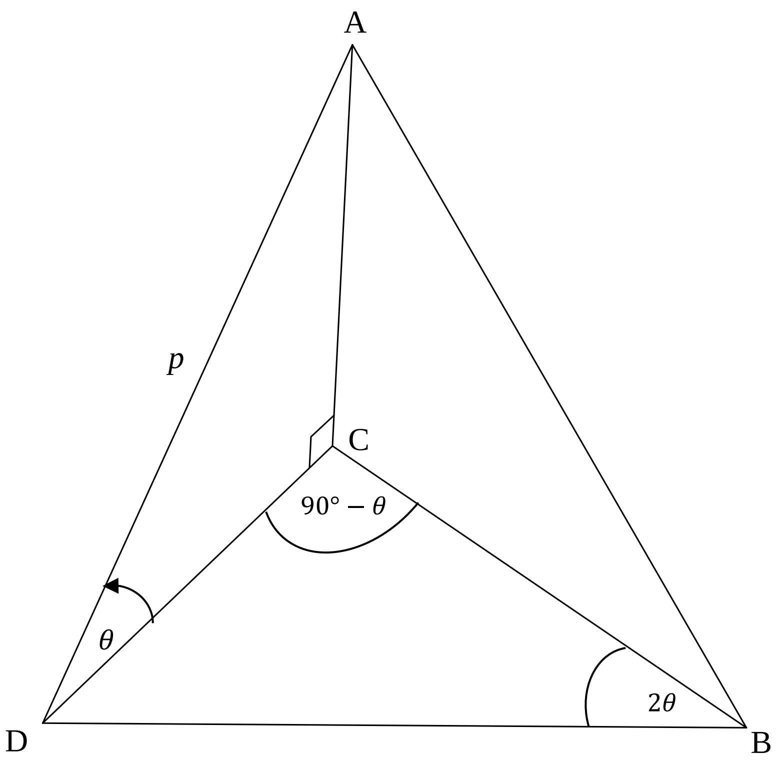
6.4 If the graph of f is shifted 30° left, give the equation of the new graph which is formed. (2)

6.5 Which transformation must the graph of g undergo to form the graph of h, where $h(x) = \sin x$? (2)

[15]

QUESTION 7

In the diagram below, D, B and C are points in the same horizontal plane. AC is a vertical pole and the length of the cable from D to the top of the pole, A, is p meters. AC ⊥ CD. $A\hat{D}C = \theta$; $D\hat{C}B = (90° - \theta)$ en $C\hat{B}D = 2\theta$.



7.1 Prove that:

$$BD = \frac{p\cos\theta}{2\sin\theta}$$ (5)

7.2 Calculate the height of the flagpole AC if $\theta = 30°$ and $p = 3$ meters. (2)

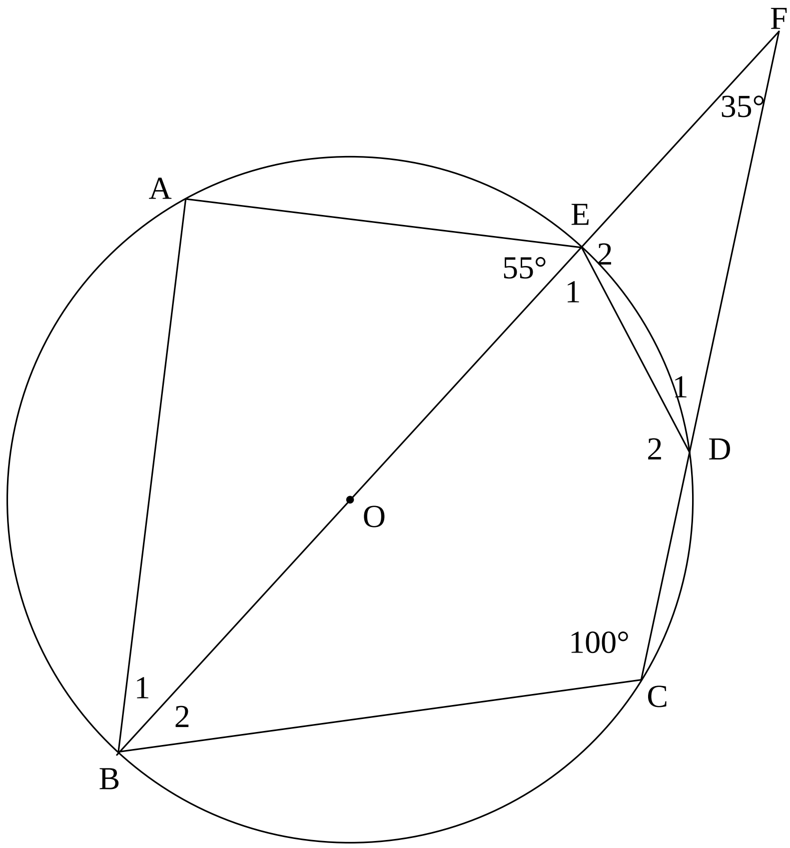
7.3 Calculate the length of the cable AB if it is further given that $A\hat{D}B = 70°$ (5)

[12]

Give reasons for all statements and calculations in QUESTION 8, 9, 10 and 11.

QUESTION 8

In the diagram, O is the centre of the circle. A, B, C, D and E are points on the circumference of the circle. Chords BE and CD produced meet at F. $\hat{C} = 100°$, $\hat{F} = 35°$ and $A\hat{E}B = 55°$



8.1 Calculate, giving reasons, the size of each of the following angles:

8.1.1 $\hat{A}$ (2)

8.1.2 $\hat{E}_1$ (2)

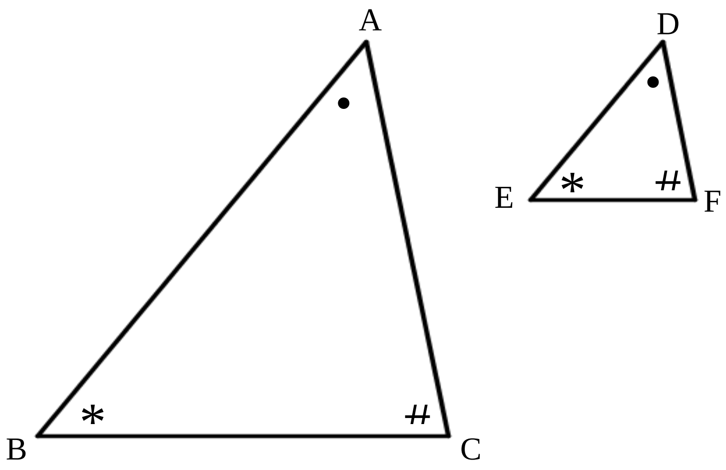
8.1.3 $\hat{D}_1$ (2)

8.2 Prove, giving reasons, that AB || CF. (4)

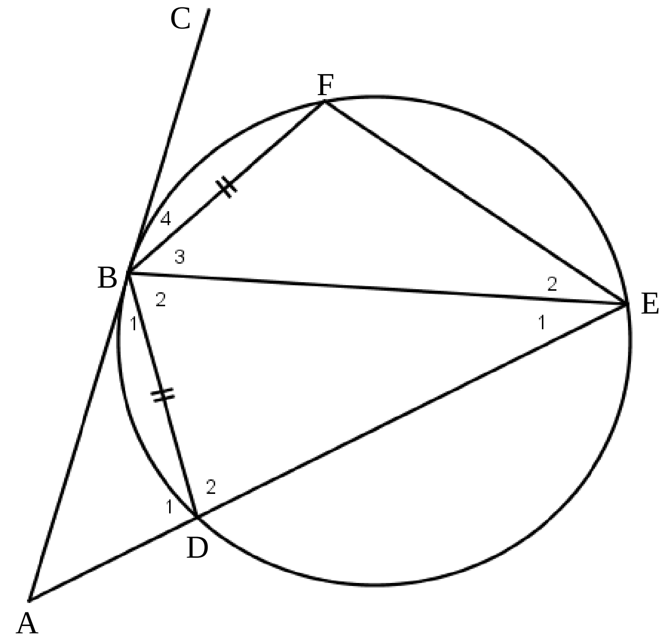
[10]

QUESTION 9

9.1 In the diagram below, ΔABC and ΔDEF are given with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Use the diagram in the ANSWER BOOK to prove the theorem that states that $\frac{DE}{AB} = \frac{DF}{AC}$. (7)



9.2 In the diagram, ABC is a tangent to the circle at B. BDEF is a cyclic quadrilateral with DB = BF. BE is drawn and ED produced meets the tangent at A.



Prove that:

9.2.1 $\hat{B}_1 = \hat{E}_2$ (3)

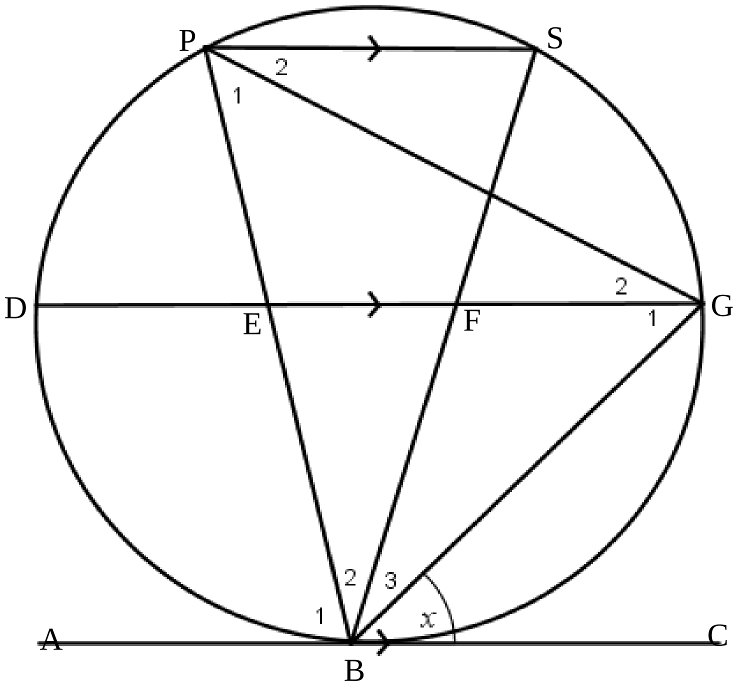
9.2.2 $\Delta BDA \mathbin{|||} \Delta EFB$ (4)

9.2.3 $BD^2 = AD.EF$ (2)

[16]

QUESTION 10

In the diagram, P, S, G, B and D are points on the circumference of the circle such that PS || DG || AC. ABC is a tangent to the circle at B. $G\hat{B}C = x$.



10.1 Give a reason why $\hat{G}_1 = x$. (1)

10.2 Prove that:

10.2.1 $BE = \frac{BP.BF}{BS}$ (2)

10.2.2 $\Delta BGP \parallel\!| \Delta BEG$ (4)

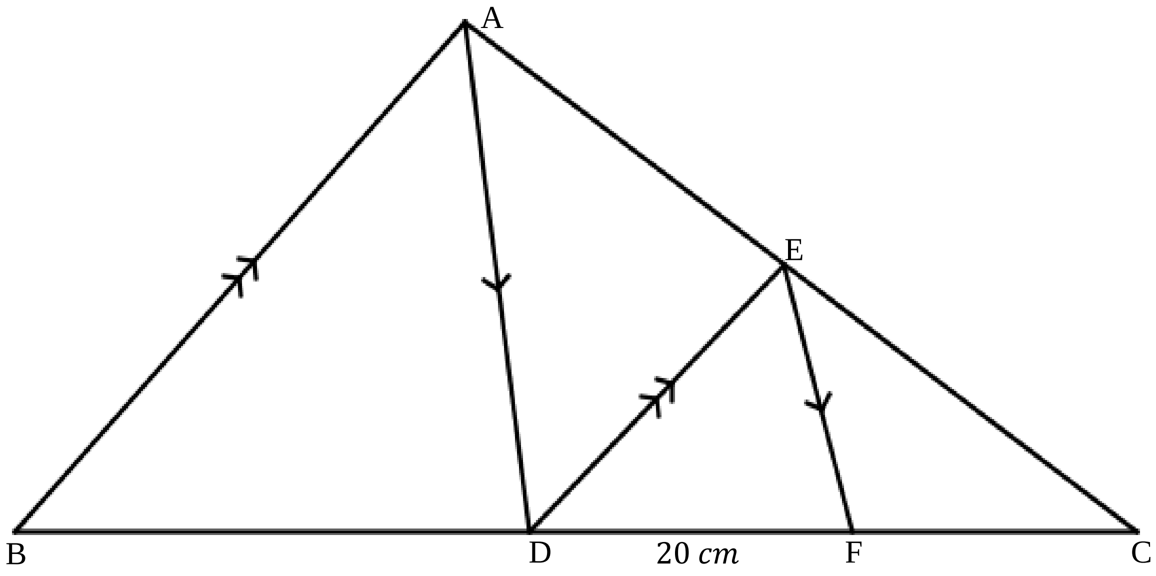
10.2.3 $\frac{BG^2}{BP^2} = \frac{BF}{BS}$ (3)

[10]

QUESTION 11

In the diagram, ∆ABC with points D and F on BC and E a point on AC such that EF || AD and

DE || BA. Further it is given that $\frac{AE}{EC} = \frac{5}{4}$ and DF = 20 cm.



11.1 Calculate, giving reasons, the length of:

11.1.1 FC (3)

11.1.2 BD (4)

11.2 Determine the following ratio:

$\frac{\text{Area } \Delta \text{ ECF}}{\text{Area } \Delta \text{ ABC}} =$ (4)

[11]

TOTAL: 150

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$T_n=ar^{n-1}$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

InΔABC: $\frac{a}{\sin \mathrm{A}}=\frac{b}{\sin \mathrm{B}}=\frac{c}{\sin \mathrm{C}}$

$$a^2=b^2+c^2-2bc.\cos A$$

$$\textit{oppervlakte}\ \Delta \mathrm{ABC}=\frac{1}{2}ab.\sin \mathrm{C}$$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$

$\mathrm{P(A)}=\frac{n(\mathrm{A})}{n(\mathrm{S})}$ P(A of B) = P(A) + P(B) – P(A en B)

$\hat{y}=a+bx$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$