Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

JUNE 2018

MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

JMATHE1

This question paper consists of 11 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet, with formulae, is included at the end of the question paper.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x, in each of the following:

1.1.1 $(x-2)(3x-1)=0$ (2)

1.1.2 $2x^2+3x-7=0$ (correct to TWO decimal places) (3)

1.1.3 $-x^2-2x+15<0$ (4)

1.1.4 $\dfrac{3^{x+1}-3^x}{3^{x-1}}=2\left(\dfrac{1}{9}\right)^{x-1}$ (5)

1.2 Solve simultaneously for x and y in the following equations:

$x-3y=1$ and $y^2+2xy-x^2=-7$ (6)

1.3 The solution(s) of a quadratic equation is given by: $x=\dfrac{n\pm\sqrt{n^2+4mn}}{2m}$

Determine the value(s) of x if the roots are equal. (5)

[25]

QUESTION 2

2.1 The following quadratic pattern is given, 15 ; 10 ; 7 ; x ; 7 ; …

2.1.1 Calculate the value of x. (3)

2.1.2 Determine the n^{th} term of the above pattern. (4)

2.1.3 Calculate the value of the 50^{th} term of the pattern. (2)

2.2 In an arithmetic series the seventh term is 34 and the fifteenth term is 74.

2.2.1 Determine the common difference of the series. (3)

2.2.2 Determine the sum of the first 40 terms of the series. (3)

2.2.3 Write the sum to 40 terms in sigma notation. (2)

2.3 A geometric series has a general term, $T_k = \dfrac{3^k}{15}$

2.3.1 Write the general term in the form $T_k = a.r^{k-1}$ (2)

2.3.2 Determine the value of n, if $\sum_{k=1}^{n}\left(\dfrac{3^k}{15}\right) = 24\dfrac{1}{5}$ (4)

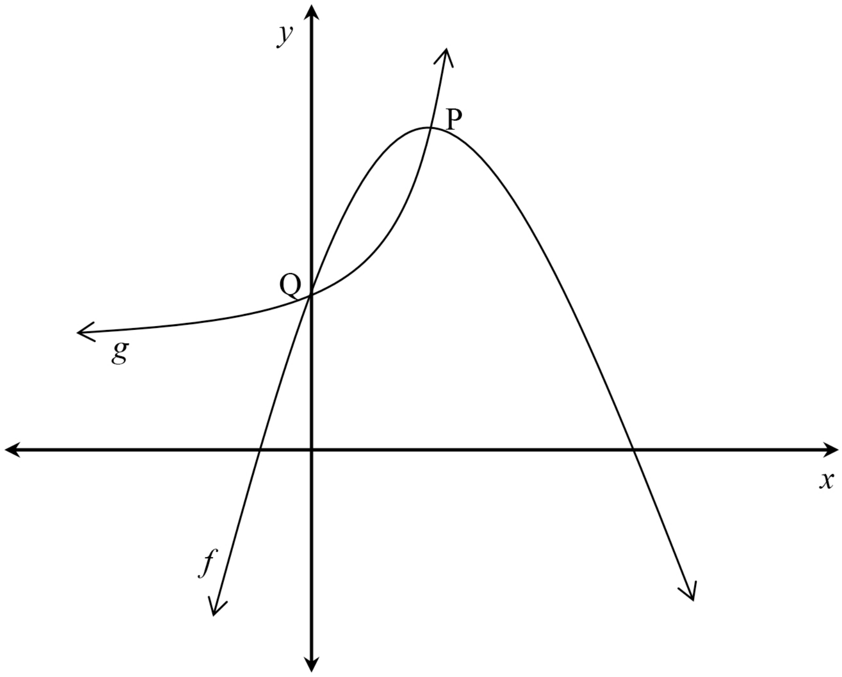
2.3.3 Is this a convergent series? Give a reason. (2)

2.4 Prove, without the use of a calculator, that $P = 9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \times \ldots$ to infinity is equal to 3. (4)

[29]

QUESTION 3

3.1 The diagram below represents the graphs of $f(x) = a(x-2)^2 + 4$ and $g(x) = b^x$
The graphs intersect at P, the turning point of f and at Q, the y-intercept of both f and g.



3.1.1 Write down the coordinates of P and Q. (2)

3.1.2 Determine the values of a and b. (4)

3.1.3 How can the domain of f be restricted such that f^{-1} may be a function? (2)

3.1.4 Determine the maximum value of $h(x) = g[f(x)]$. (2)

3.2 Consider the following two functions: $p(x) = x^2 + 1$ and $r(x) = x^2 + 2x$

3.2.1 Write down the range of p. (1)

3.2.2 Describe the transformation from p to r. (3)

[14]

QUESTION 4

Given the equation of f, a hyperbola, $f(x)=\dfrac{-3}{x+1}+5$, answer the questions that follow.

4.1 Calculate the y-intercept of f. (1)

4.2 Calculate the x-intercept(s) of f. (2)

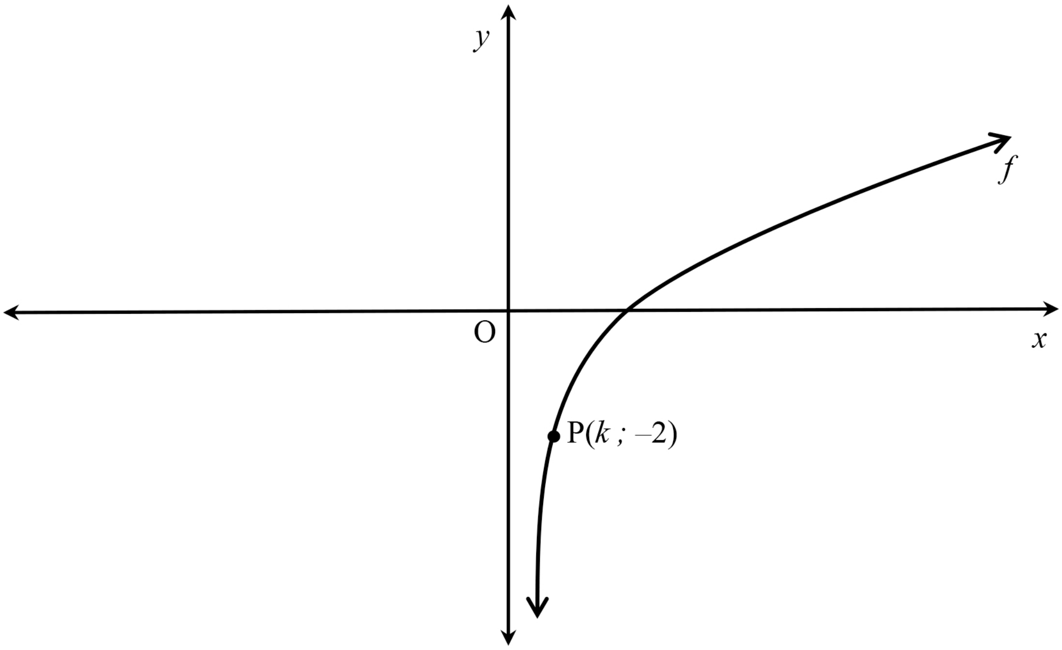
4.3 Sketch the graph of f, clearly indicating the asymptotes and intercepts with the axes. (3)

4.4 Write down the equation of the graph formed if the graph of f is shifted 3 units to the right and then reflected across the x-axis. (3)

[9]

QUESTION 5

The diagram represents a sketch graph of the function, $f(x)=\log_3 x$, with P(k ; – 2) a point on the curve.



5.1 Determine the equation of f^{-1} in the form $f^{-1}(x)=...$ (2)

5.2 Explain how you would use the graph of f to sketch the graph of f^{-1}. (2)

5.3 Find the value of k. (2)

5.4 Hence, or otherwise, solve for x, if $\log_3 x<-2$ (2)

5.5 For which value(s) of x, will $f(x).f'(x)\geq 0$? (2)

[10]

QUESTION 6

6.1 Interest on a credit card is quoted as 23% p.a. compounded monthly. What is the effective annual interest rate? Give your answer correct to two decimal places. (3)

6.2 Mary has just been to the bank to withdraw all her savings, and to her delight she discovers she has saved R15 768,39 over the last 10 years. If the interest was calculated at 4,38% per annum, compounded quarterly, how much money did Mary originally invest at the bank? (3)

6.3 A company purchased a new vehicle for R200 000. The vehicle's value depreciated to R50 710,00 at a rate of 24% p.a. on a reducing balance. The company wants to replace the vehicle with a new one. The replacement cost of the vehicle increased by 18% p.a. compounded annually, calculate:

6.3.1 How long it took for the vehicle's value to depreciate to R50 710,00 (4)

6.3.2 The cost of a new vehicle to replace the old one (2)

6.3.3 The total amount required, if the old vehicle is sold and the proceeds contribute towards the new vehicle (1)

[13]

QUESTION 7

7.1 Determine the derivative of $f(x) = 1 - 3x^2$ from first principles. (4)

7.2 Determine $\frac{dy}{dx}$ if $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ (3)

7.3 Determine the coordinates of the point on the graph of $y = 3x^2 - 2x + 1$ where the gradient is 4. (4)

[11]

QUESTION 8

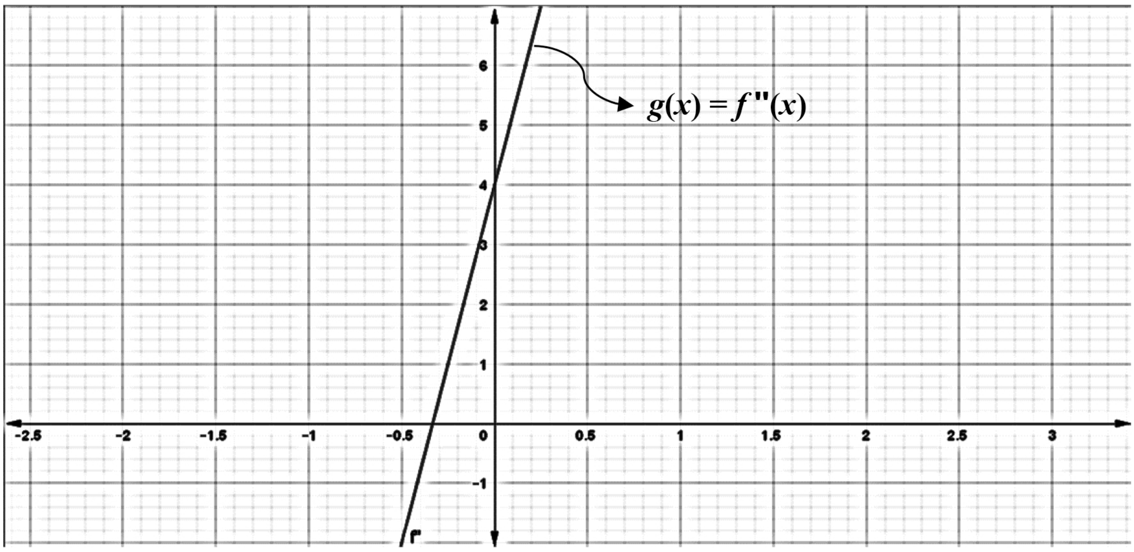
8.1 The function, $f(x) = ax^3 + bx^2 + cx + d$, represents a cubic graph. The x-intercepts of the graph are $(-2;0)$, $\left(\frac{2}{3};0\right)$ and $(3;0)$. The points $P(x;y)$ and $Q(2;-16)$ are the turning points of f.

8.1.1 Show that the equation of f is given by $f(x) = 3x^3 - 5x^2 - 16x + 12$. (5)

8.1.2 Determine the coordinates of P, the local maximum of f. (4)

8.1.3 Draw the graph of f, clearly indicating the turning points and the intercepts with the axes. (4)

8.2 Given: $f(x) = ax^3 + bx^2 + 3x + 3$ and $g(x) = f''(x)$ where $g(x) = 12x + 4$



8.2.1 Determine the values of a and b. (2)

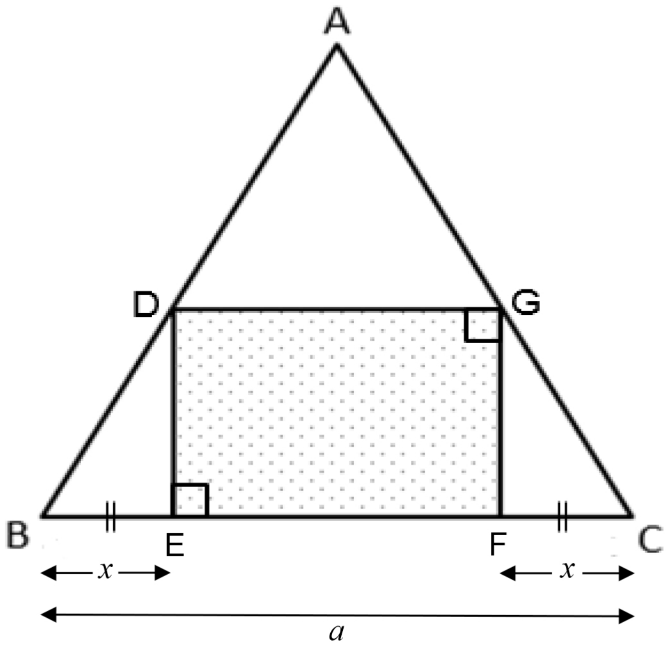
8.2.2 For which value(s) of x, will $f'(x)$ be increasing? (2)

8.2.3 Describe the concavity of f for all $x \in R$ (2)

[19]

QUESTION 9

In the sketch Δ ABC is an equilateral triangle with BC = ***a*** units.
DEFG is a rectangle. BE = FC = ***x*** units.



9.1 Prove that the area of the rectangle, $A = \sqrt{3}ax - 2\sqrt{3}x^2$ (4)

9.2 Determine, in terms of ***a***, the maximum area of the rectangle. (5)

[9]

QUESTION 10

10.1 The sport teacher at a school analysed data to determine how many learners play sport, as well as the gender of each learner. The data is presented in the following table.

	Do not play sport (not S)	Play sport (S)	Total
Male (M)	51	69	120
Female (F)	49	67	116
Total	100	136	236

10.1.1 Determine the probability that a learner selected at random is female and plays sport. (1)

10.1.2 Are the events "male" and "do not play sport" independent? Show ALL calculations to support your answer. (4)

10.2 In a bag, there are x blue balls and 2 red balls. A ball is selected at random, the colour is recorded and then replaced. Another ball is then selected at random, the colour is recorded and then replaced. The probability that the two balls are different colours is 0,375.

10.2.1 Draw a tree diagram of the above scenario. (3)

10.2.2 Hence, determine the value of x. (3)

[11]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \partial^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ $\quad P(A) = \frac{n(A)}{n(S)}$ $\quad P\ (A \text{ or } B) = P\ (A) + P\ (B) - P$

$(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$