Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2015

MATHEMATICS P2
MEMORANDUM

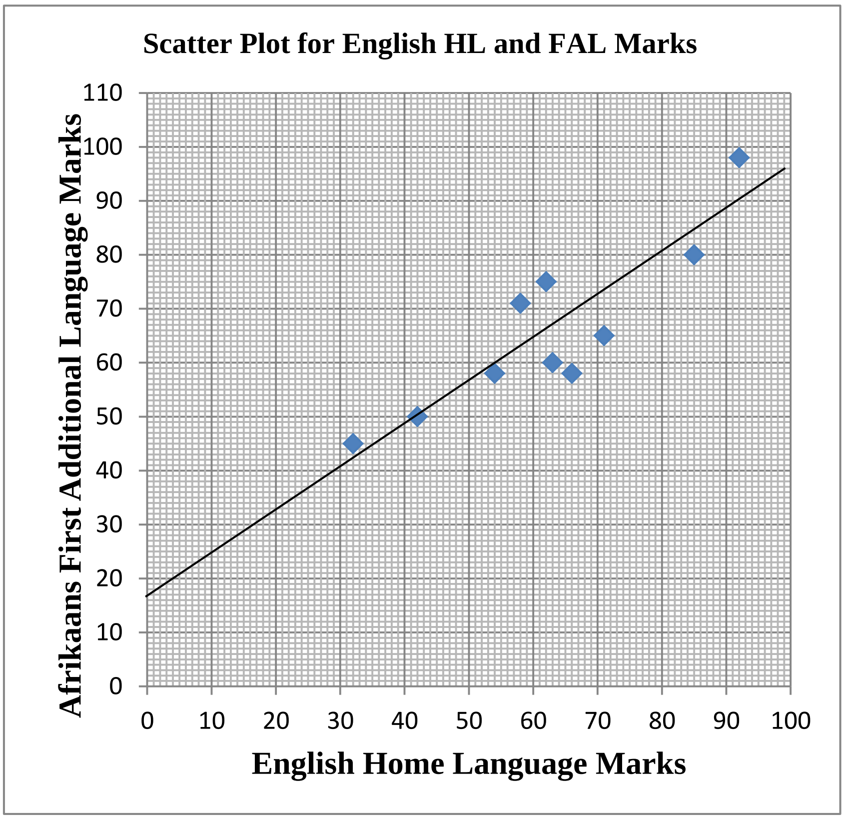
MARKS: 150

This memorandum consists of 14 pages.

QUESTION 1

1.1

English HL	42	54	85	32	63	71	92	62	58	66
Afrikaans FAL	50	58	80	45	60	65	98	75	71	58



✓✓✓ plotting all ten points

✓regression line

(4)

1.2	$a = 18{,}03563\ldots$ $b = 0{,}76\ 7429\ldots$ $y = 18{,}04 + 0{,}77x$	✓ value for A ✓value for B ✓equation	(3)
1.3	$r = 0{,}88$	✓✓Answer	(2)
1.4	Strong positive correlation. OR Linear trend OR If learner got high marks in English then they would achieve similar marks in Afrikaans	✓Answer	(1)
1.5	74% Accept answers from 73 to 75	✓✓Answer	(2)
			[12]

QUESTION 2

2.1	$\bar{x} = \frac{\sum x}{n}$ $= \frac{1155}{20}$ $= 57,75$ $\sigma = (6,737 \ldots.)^2$ $= 45,39$ Answer only: Full Marks	✓ Answer ✓ square of variance ✓ Answer	(3)
2.2.1	22 students	✓ Answer	(1)
2.2.2	$\bar{x} = \frac{\sum x}{n}$ $= \frac{1320}{22}$ $= 60$ Answer only: Full Marks	✓ substitution ✓ Answer	(2)
2.2.3	$\sigma = \sqrt{\frac{1012}{22}}$ $= 6,782$ Answer only: Full Marks	✓substitution ✓ Answer	(2)
2.3	$\frac{1155 + 5x}{25} = 60$ $1155 + 5x = 1500$ $5x = 345$ $x = 69$ Each boy must be 69 kg	✓ $\frac{1155 + 5x}{25} = 60$ ✓Answer	(2)
			[10]

QUESTION 3

3.1.1	E = $\left[\frac{x_1+x_2}{2};\ \frac{y_1+y_2}{2}\right]$ = $\left[\frac{3+5}{2};\ \frac{5+3}{2}\right]$ E = (4; 4)	✓substitution into correct formula ✓ co-ordinates of E.	(2)
3.1.2	$m_{\text{AB}} = m_{\text{DC}}$ $\frac{y-y_1}{x-x_1} = \frac{3-1}{5+1}$ $\frac{y-5}{x-3} = \frac{2}{6}$ $\therefore y-5=2\ \&\ x-3=6$ $\therefore y=7\ \&\ x=9$ B(9; 7) OR $\frac{-1+x}{x} = 4\ ,\quad \frac{1+y}{x} = 4$ $x = 9$, $y = 7$ B(9; 7)	✓equating two gradients. ✓ simplification ✓co-ordinates for B ✓$\frac{-1+x}{x} = 4$ ✓ $\frac{1+y}{x} = 4$ ✓co-ordinates for B	(3)

3.1.3	F = $\frac{-1+5}{2};\ \frac{1+3}{2}$ F = [2; 2] $y - y_1 = m(x - x_1)$ $y-2 = 1(x-2)$ $y = x-2+2$ $y = x$	✓ co-ordinates of F ✓ formula ✓ correct value of m ✓correct substitution into formula ✓ Answer	(5)
3.2	$m_{\text{DE}} = m_{\text{EG}}$ $\frac{2,5-4}{t+1-4} = \frac{3}{5}$ $\frac{-1,5}{t-3} = \frac{3}{5}$ $3t - 9 = -7,5$ $3t = 1,5$ $t = 0,5$	✓ equating gradients ✓ correct substitution ✓ simplification ✓ answer	(4)

3.3	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(9-3)^2 + (7-5)^2}$ $= \sqrt{40}$ $= \sqrt{(-1-3)^2 + (1-5)^2}$ $= \sqrt{32}$ ∴ ABCD is NOT a rhombus because AB ≠AD OR $m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-3}{3-5}$ $= -\frac{2}{3}$ $m_{DB} = \frac{3}{5}$ $m_{AC} \times m_{DB} = -\frac{2}{3} \times \frac{3}{5}$ $= -\frac{2}{5}$ $\neq -1$ ∴ ABCD is not a rhombus because: $m_{AC} \times m_{DB} \neq -1$	✓ substitution in formula ✓ answer ✓ answer ✓ statement ✓ reason ✓ substitution in formula ✓ answer ✓ answer ✓ statement ✓ reason	(5)
			[19]

QUESTION 4

4.1	$r^2 = (3+1)^2 + (1-4)^2$ $= 16 + 9$ $r^2 = 25$ $r = 5$ $(x-a)^2 + (y-b)^2 = r^2$ $(x+1)^2 + (y-4)^2 = 25$	✓ use of distance formula ✓ $r^2 = 25$ ✓✓ substituting values into formula	(4)
4.2	$N\hat{Q}M = 90°$ [angle subtended by diameter]	✓answer	(1)
4.3	$(m_{NQ}).(m_{QM}) - 1$ $\frac{y-7}{x+5} \times \frac{y-1}{x-3} = -1$ $\frac{0-7}{x+5} \times \frac{0-1}{x-3} = -1$ $\frac{7}{x+5} \times \frac{-1}{x+5} = -1$ $x^2 + 2x - 15 = -7$ $x^2 + 2x - 8 = 0$ $(x+4)(x-2) = 0$ $x = -4, x = 2$ Q(-4; 0)	✓substitution into formula for perpendicular gradients ✓ simplification and solving trinomial ✓ coordinates of Q	(3)
4.4	$m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{7-1}{-5-3}$ $= \frac{6}{-8} = -\frac{3}{4}$	✓substitution in formula ✓answer	(2)
4.5	$m_{MN} = -\frac{3}{4}$ $\tan\theta = -\frac{3}{4}$ $\theta = 180° - 36{,}86°$ $\theta = 143{,}13°$ $m_{QM} = \frac{1-0}{3+4}$ $\tan\beta = \frac{1}{7}$ $\beta = 8{,}13°$ $\alpha = 143{,}13° - 98{,}13°$ $\alpha = 45°$	✓ Gradient of MN ✓value of θ ✓ Gradient of QM ✓ value of β ✓ value of α	(5)

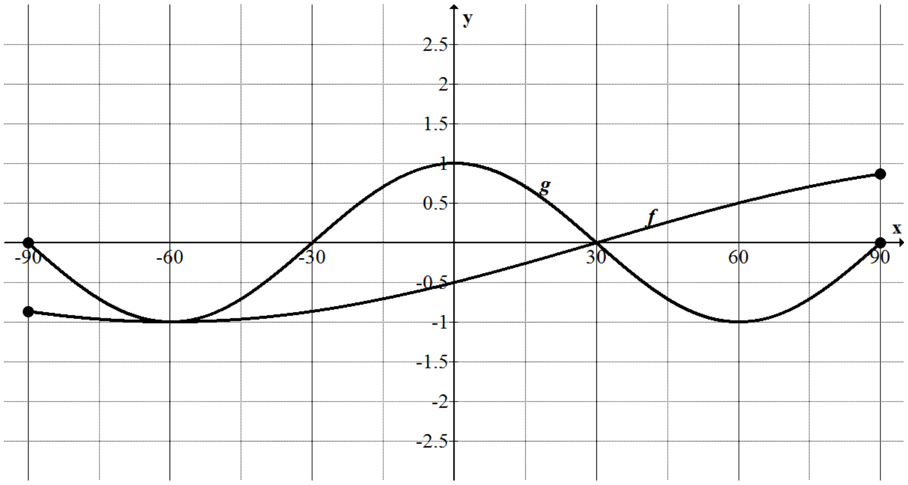
4.6	$m_{MP} = \frac{4-1}{-1-3}$ $= -\frac{3}{4}$ $m_{tan} = \frac{4}{3}$ $y - y_1 = m(x - x_1)$ $y - 1 = \frac{4}{3}(x-3)$ $y = \frac{4}{3}x - 4 + 1$ $y = \frac{4}{3}x - 3$	✓ m_{MP} ✓m_{tan} ✓substitution in formula ✓simplification ✓answer	(5)
			[20]

QUESTION 5

5.1	$\cos 75° + \cos 15° = \cos(45° + 30°) + \cos(45° - 30°)$ $= 2\cos 45°.\cos 30°$ $= 2\left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{6}}{2}$	✓expression in terms of 30° and 45^0 ✓ use of compound identity ✓substitution of values ✓ answer.	(4)
5.2	$1 + 4\sin^2 A - 5\sin A + \cos 2A = 0$ $1 + 4\sin^2 A - 5\sin A + 1 - 2\sin^2 A = 0$ $2\sin^2 A - 5\sin A + 2 = 0$ $(2\sin A - 1)(\sin A - 2) = 0$ $2\sin A = 1$ or $\sin A = 2$ (no solution) $\sin A = \frac{1}{2}$ $A = 30° + k.360$ $A = 150° + k.360°; k \in z$	✓ cos 2A in terms of Sin A ✓standard form ✓ factors ✓ for each value of sin A ✓gen. sol in 1st quadrant ✓gen. sol in 2nd quadrant ✓gen. sol notation	(7)

5.3	$\frac{\sin 2A}{1+\cos 2A} = \tan A$ LHS= $\frac{2\sin A.\cos A}{1+2\cos^2 A-1}$ $= \frac{2\sin A.\cos A}{2\cos^2 A}$ $= \frac{\sin A}{\cos A}$ $= \tan A$ LHS = RHS	✓ double angle identity for sin2A ✓ double angle identity for cos 2A ✓ LHS = RHS	(3)
5.4	$\frac{\cos x \tan x \sin 23° \cos 23°}{\sin\ 46°(-\sin x)}$ $\frac{\cos x.\frac{\sin x}{\cos x}.\sin 23°.\cos 23}{-2\sin 23° \cos 23°\ .\sin x}$ $-\frac{1}{2}$	✓$\cos x$ ✓$\tan x$ ✓$\sin\ 46°$ ✓$-\sin x$ ✓$\frac{\sin x}{\cos x}$ ✓$2\sin 23° \cos 23°$	(6)
			[20]

QUESTION 6

6.1	120°	✓ answer.	(1)
6.2		*f* ✓ *x*-int ✓ *y*-int ✓ turning point *g* ✓ *x*-int ✓*y*-int ✓ turning point	(6)
6.3	6.3.1 $x \in (30° ; 90]$	✓ both values correct ✓ correct notation	(2)
	6.3.2 $x \in (-90°;\ -30°)$	✓ both values correct ✓ correct notation	(2)
6.4	$-4 \leq y \leq 0{,}5$ OR [-4 ; 0,5]	✓interval ✓ values	(2)
			[13]

QUESTION 7

7.1	$In\ \Delta\ CBE$ $\frac{CE}{BC} = \sin C\hat{B}E$ BC = $\frac{CE}{\sin \hat{B}}$ $= \frac{3}{\sin 16{,}7°}$ BC = 10,44	✓ ratio for sin ✓ simplification and substitution ✓ Answer	(3)
7.2	$In\ \Delta\ ABC$ $\frac{AB}{\sin \hat{C}} = \frac{BC}{\sin A}$ AB = $\frac{BC\ .\sin \hat{C}}{\sin \hat{A}}$ $AB = \frac{10{,}44.\sin 32{,}3°}{\sin 18{,}1}$ AB = $17{,}96^0$	✓ use of sine rule ✓ simplification and substitution ✓ answer	(3)
7.3	$\frac{AD}{AB} = \sin C\hat{B}D$ $AD = AB.\sin C\hat{B}D$ $= 17{,}96\ \ \sin 33{,}7°$ AD = 9,96m AD = 10m	✓ ratio for sin ✓ simplification & substitution ✓ answer	(3)
			[9]

QUESTION 8

8.1.1	$\hat{T}_2 = x$ [OS = OT, radii] $\hat{O}_1 = 180° - 2x$ [sum of ∠'s of Δ]	✓S/R ✓ S/R	(2)
8.1.2	$\hat{P} = \frac{\hat{O}_1}{2}$ [∠ at centre = 2×∠at circumference] $= \frac{180-2x}{2}$ $= 90° - x$	✓ statement and reason ✓answer	(2)
8.1.3	$\hat{O}_1 = \hat{R}$ [SOQR is parallelogram]or [opp angles of parallelogram] $\hat{R} = 180° - 2x$	✓ statement & reason ✓answer	(2)
8.2	$\hat{O}_1 = \hat{R}$ [opp angles of parallelogram] $180° - 2x = 90° + x$ $3x = 90°$ $x = 30°$	✓ statement and reason ✓substitution of values ✓ Answer	(3)
			[9]

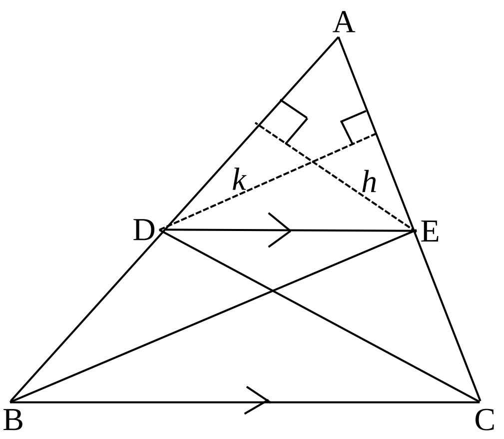
QUESTION 9

9.1	$\hat{M}_1 = \hat{M}_2 = 90°$ [line from centre to midpoint of chord] $\hat{M}_2 = \hat{A}$ [given QA⊥TA] $\hat{M}_2 + \hat{A} = 180°$ ∴ MTAR is a cyclic quad [opp ∠'s are supplementary]	✓$M_2 = 90°$ ✓ reason ✓M_2 = A & reason ✓conclusion and reason	(4)
9.2	In ΔPMR & ΔTMR (i) $M_1 = M_2$ [OR bisects PT] (ii) PM = MT [M is midpoint of PT] (iii) MR = MR [common] ∴ ΔPMR ≡ ΔTMR [S A S] ∴ PR = TR	✓ statement and reason ✓statement and reason ✓ statement and reason ✓ SAS ✓conclusion	(5)
9.3	$\hat{T}_2 = \hat{P}$ [Tan chord theorem] $\hat{T}_1 = \hat{T}_2$ [Both equal to P]	✓ $\hat{T}_2 = \hat{P}$ ✓ Tan chord ✓ conclusion	(3)
			[12]

QUESTION 10

10.1	Proportional	✓ Answer	(1)
10.2.1	$A_4 = D_1$ [tan chord theorem] $D_1 = x$ $D_1 = E_2$ [angles in same segment] $E_2 = x$ $A_4 = C_2$ [alt int.]	✓statement and reason ✓statement and reason ✓statement and reason	(3)
10.2.2	In ΔACF &ΔADC i) $A_3 = A_3$ [common] ii) $C_2 = D_1$ ΔACF /// ΔADC [equiangular]	✓statement and reason ✓statement and reason ✓statement and reason	(3)
10.2.3	ΔACF /// ΔADC $\frac{AC}{AD} = \frac{AF}{AC} = \frac{CF}{DC}$ $\frac{AC}{AD} = \frac{AF}{AC}$ $AF = \frac{AC.AC}{AD}$ But AC = AO $AF = \frac{AO^2}{AD}$	✓ sides in proportion ✓ choosing correct proportion ✓ simplification ✓ answer	(4)
			[11]

QUESTION 11

11.1	 RTP: $\frac{AD}{DB}=\frac{AE}{EC}$ Constr: Draw height *h* from E to AD Draw height *k* from D to AE. Join BE and DC PROOF: $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{ BDE}}=\frac{\frac{1}{2}.\text{AD}.h}{\frac{1}{2}.\text{DB}.h}$ $=\frac{\text{AD}}{\text{BD}}$ $\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{CED}}=\frac{\frac{1}{2}.\text{AE}.k}{\frac{1}{2}.\text{EC}.k}$ $=\frac{\text{AE}}{\text{EC}}$ But Area ΔBDE = Area Δ CED (same base and same height) $\therefore\ \frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{BDE}}=\frac{\text{area }\Delta\text{ADE}}{\text{area }\Delta\text{ CED}}$ $\therefore\ \frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$	✓ constr ✓ ratio of area of ΔADE: ΔBDE ✓ $\frac{\text{AD}}{\text{BD}}$ ✓ratio of area of ΔADE : ΔCED ✓ $\frac{\text{AE}}{\text{EC}}$ ✓equating two areas ✓conclusion $\frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$	(6)

11.2.1	$\frac{AD}{DB} = \frac{AM}{MN} = \frac{3}{2}$ [Prop. theorem; DE ‖ BC]	✓ answer ✓ reason	(2)
11.2.2	In ΔADE & ΔABC i) $\hat{A} = \hat{A}$ [common] ii) $A\hat{D}E = A\hat{B}C$ [corresponding; DE ‖ BC] ∴ ΔADE/// Δ ABC [equiangular] $\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\frac{AD}{BC} = \frac{DE}{BC} = \frac{3}{2}$	✓ Corresponding angles equal ✓showing ΔADE|||ΔABC ✓ sides in proportion ✓ answer	(4)
11.2.3	$\frac{\text{area } \Delta ADE}{\text{area } \Delta ABC} = \frac{\frac{1}{2}DE.AM}{\frac{1}{2}BC.AN}$ $= \frac{3\times3}{5\times5}$ $= \frac{9}{25}$	✓Ratio of areas ✓substitution of values in denominator and numerator ✓simplification and answer	(3)
			[15]
	TOTAL:		**150**