Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2018

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

JMATHE2

This question paper consists of 14 pages, including a 1 page information sheet and a special answer book.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary round off your answers to TWO decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A class of 15 learners was given a test out of 100. The marks obtained by the learners are as follows:

34 58 26 44 28 29 36 49 54 43 45 59 37 29 48

1.1 Calculate the mean mark for these learners. (2)

1.2 Calculate the standard deviation. (2)

1.3 How many learners got marks that are within one standard deviation of the mean? (3)

1.4 Calculate the semi-IQR. (3)
[10]

QUESTION 2

A group of 64 learners wrote an English essay and the time taken to complete the task was recorded as follows:

Time (in minutes)	**Frequency**	**Cumulative frequency**
$5 \leq t < 10$	3	
$10 \leq t < 15$	5	
$15 \leq t < 20$	y	
$20 \leq t < 25$	16	
$25 \leq t < 30$	15	
$30 \leq t < 35$	17	
$35 \leq t < 40$	y	

2.1 Calculate the value of y. (2)

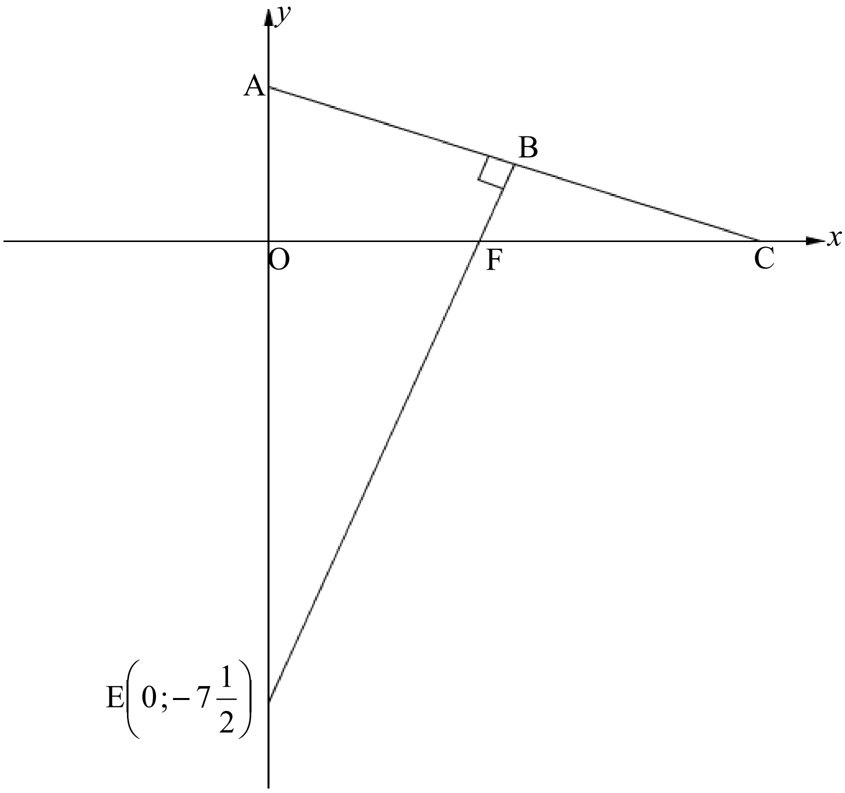
2.2 Complete the cumulative frequency table. (2)

2.3 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (3)

2.4 Use your graph to estimate the number of learners who completed the task after 33 minutes. (3)
[10]

QUESTION 3

In the diagram below, the straight line AC is drawn having A and C, y- and x- intercepts respectively. The equation of AC is $x + py = p$, $p > 0$. It is also given that $OC = 4OA$. The straight line EB is drawn. B is the point on AC such that $EB \perp AC$. EB cuts the y-axis at $E\left(0;-7\frac{1}{2}\right)$ and x-axis at F.



3.1 Calculate the coordinates of A. (2)

3.2 Calculate the value of p. (4)

3.3 Determine the equation of EB in the form $y = mx + c$ (2)

3.4 Calculate the coordinates of B. (4)

3.5 Calculate the coordinates of F. (2)

3.6 Calculate area of quadrilateral AOFB. (5)

3.7 Write down the length of the radius of the circle passing through F, B and C. (1)

3.8 Hence, write down the equation of the circle passing through F, B and C if the coordinates of the midpoint of FC are $\left(\frac{47}{16};0\right)$. (2)

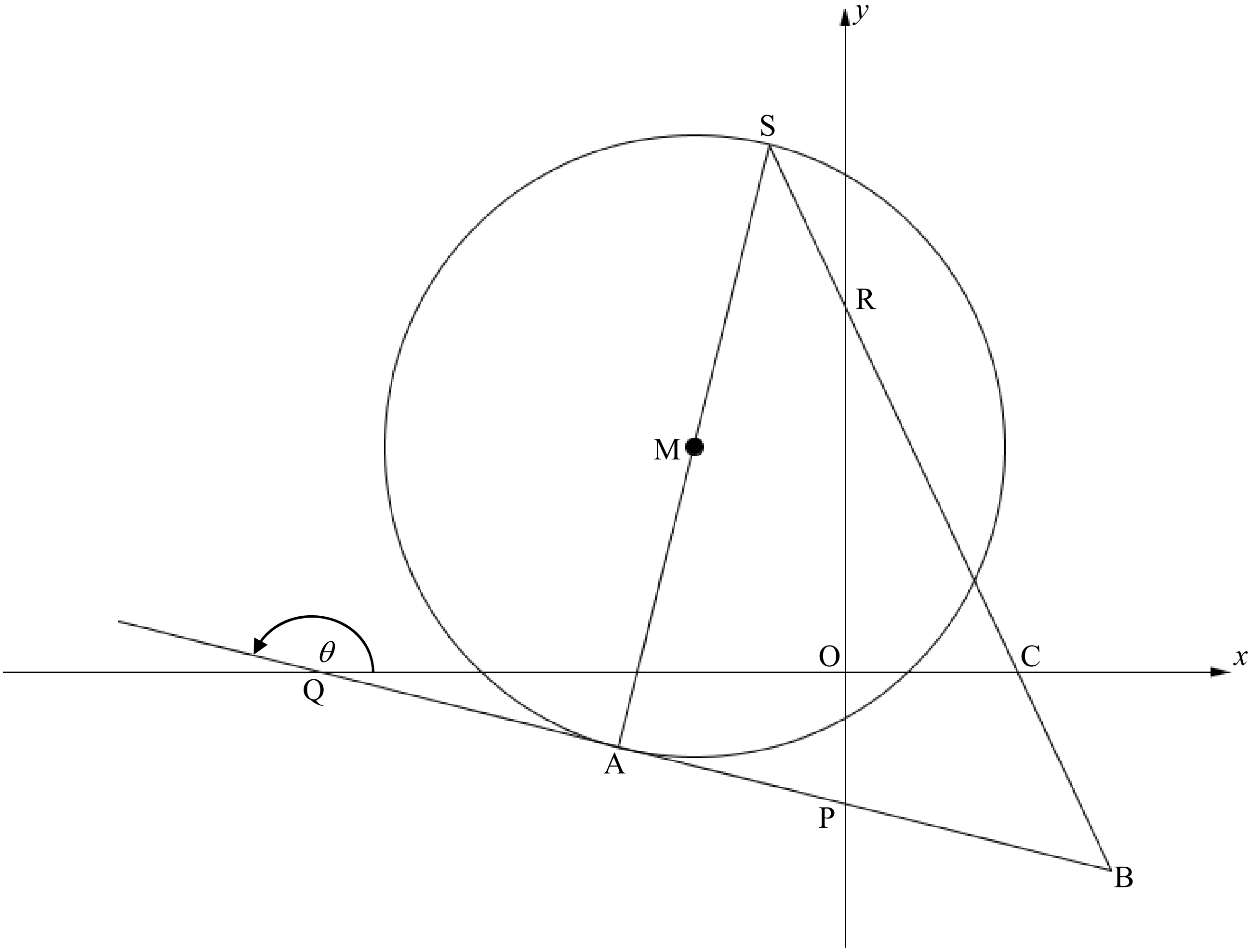
[22]

QUESTION 4

In the diagram below, the equation of the circle with centre M is $x^2 + y^2 + 6x - 8y - 1 = 0$.

AS is the diameter of the circle. The equation of the tangent to the circle at A is $y = -\frac{1}{5}x + k$

Line SRCB is drawn.



4.1 Determine the coordinates of M. (3)

4.2 Write down the length of the radius of the circle. (Leave your answer in simple surd form.) (1)

4.3 Determine the equation of the diameter AS. (3)

4.4 Determine the coordinates of A. (6)

4.5 Hence or otherwise, calculate the value of k. (2)

4.6 If it is further given that $O\hat{R}C = 11^0$, prove that RCPQ is a cyclic quadrilateral. (Round off your answer to the nearest degree.) (4)

[19]

QUESTION 5

5.1 Given that $2\sin 27^0 = t$, express each of the following in terms of *t*:

5.1.1 $\sin 54^0$ (4)

5.1.2 $\tan 513^0.\cos 27^0$ (3)

5.1.3 $\cos 87^0$ (4)

5.2 Simplify, without using a calculator:

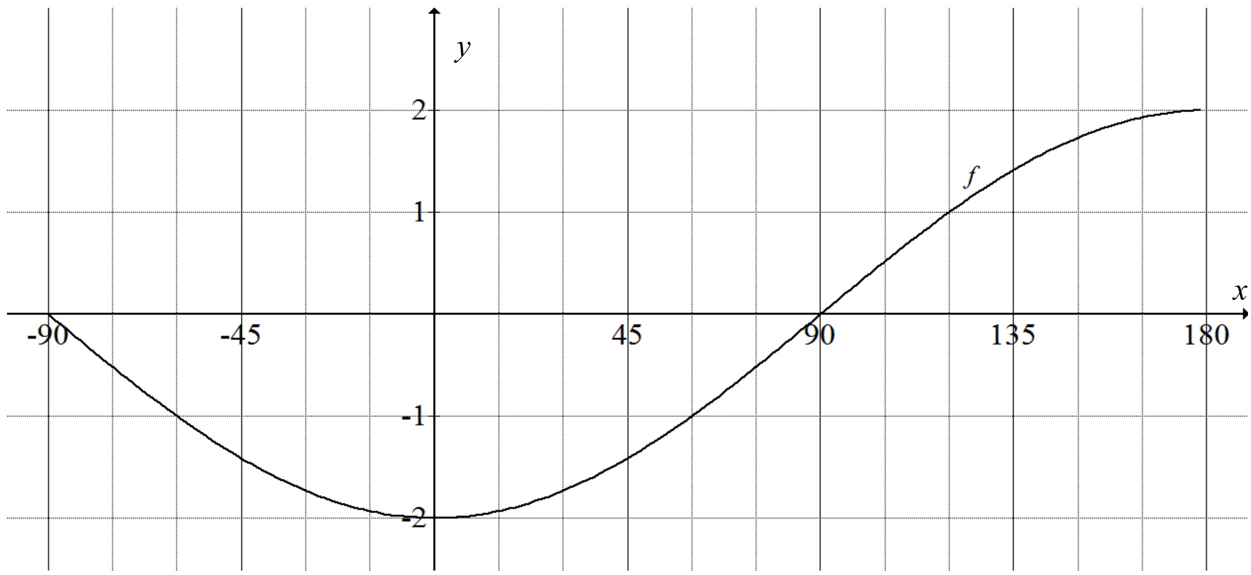
$$\frac{\sin(-2\alpha)\cos(90^0+\alpha)}{\sin(-\alpha+360^0)\cos(-\alpha-180^0)}$$ (6)

5.3 Determine the general solution of the equation: $9\sin^2 x - 4\cos^2 x = 0$ (5)

[22]

QUESTION 6

In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for the interval $-90^0 \leq x \leq 180^0$



6.1 Write down the amplitude of f. (1)

6.2 Write down the range of $f(x) + 3$. (2)

6.3 On the same system of axes draw the graph of $g(x) = \sin(x + 30^0)$. (3)

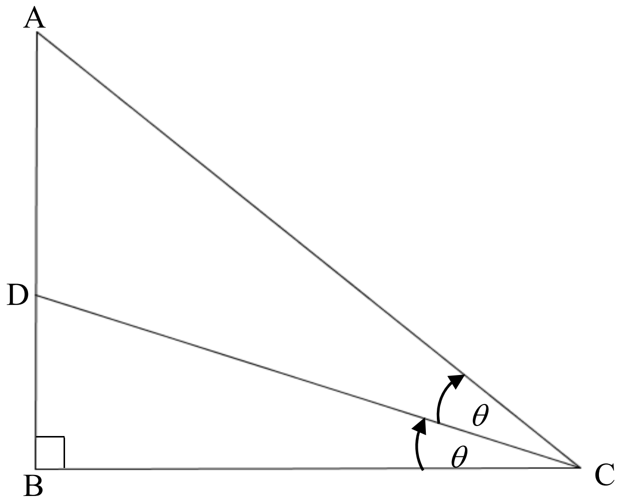
6.4 Determine the values of x in the interval $-90^0 \leq x \leq 90^0$ for which $x.g(x) < 0$? (2)

6.5 Write the equation of h, where h is formed by shifting g, 60^0 to the left and 2 units downwards. (Leave your answer in simplified form.) (3)

[11]

QUESTION 7

In the diagram below, ABC is a right angled-triangle with $\hat{B} = 90^0$. The straight line CD bisects $A\hat{C}B$ and cuts AB at D. $D\hat{C}B = \theta$.



7.1 Write down the size of $\hat{A}$ in terms of θ. (1)

7.2 Write down the ratio of $\sin\theta$. (1)

7.3 If it is further given that $\frac{DB}{AD} = \frac{1}{2}$ hence, or otherwise, show that $2\cos 2\theta - 1 = 0$. (5)

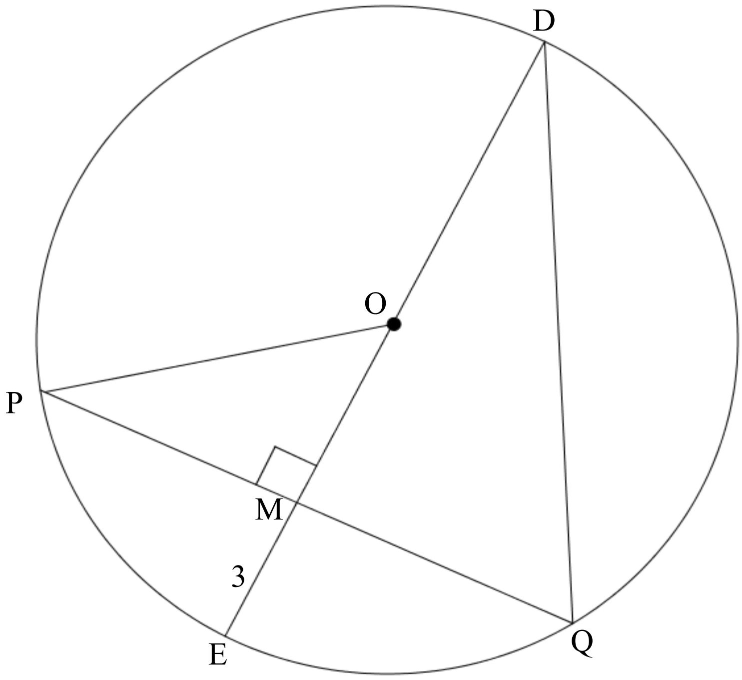
[7]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

8.1 Complete:
A line drawn from the centre of the circle to the midpoint of the chord is … (1)

8.2 In the diagram drawn below, O is the centre of the circle. PQ is a chord and DE is the diameter. PQ = 12 units, DM = $2x$ and ME = 3 units.



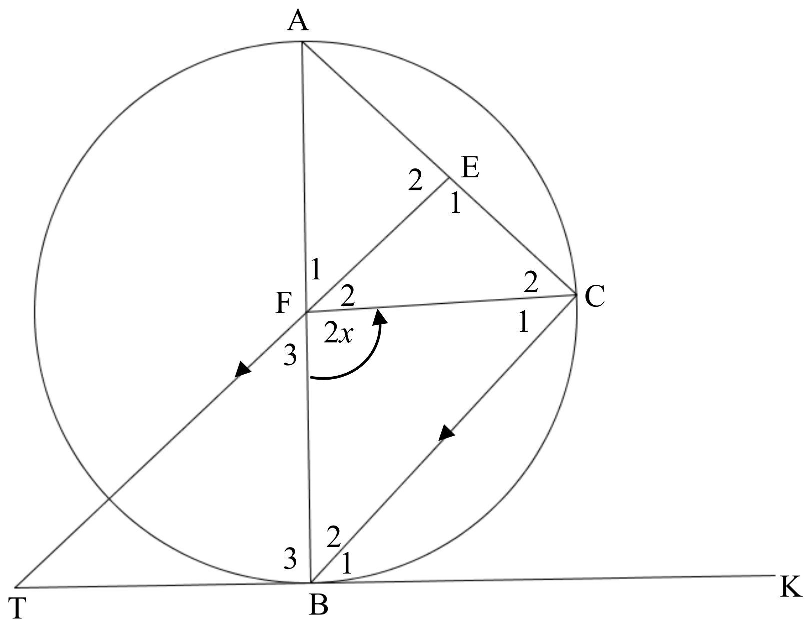
8.2.1 Write down the length of MO in terms of x. (1)

8.2.2 Calculate the value of x. (4)

8.2.3 Hence, calculate the length of DQ. (Leave your answer in simplest surd form.) (3)
[9]

QUESTION 9

In the diagram below, F is the centre of the circle. TBK is a tangent to the circle at B. E and F are points on AC and AB respectively. EF is produced to T. $BC \parallel TE$ and $B\hat{F}C = 2x$.

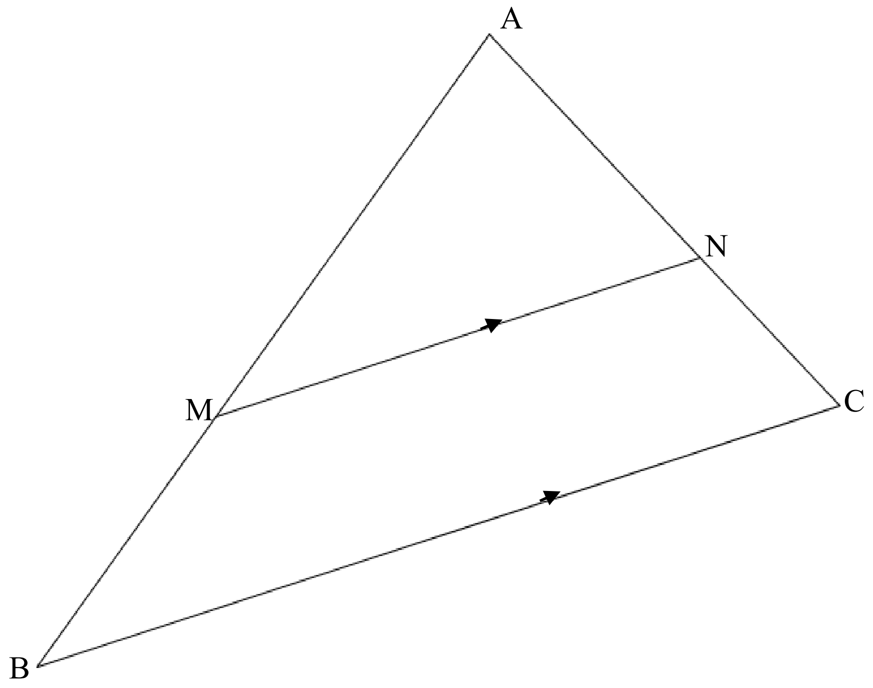


9.1 Name with reasons, four angles each equal to x. (8)

9.2 Prove that ATBE is a cyclic quadrilateral. (2)
[10]

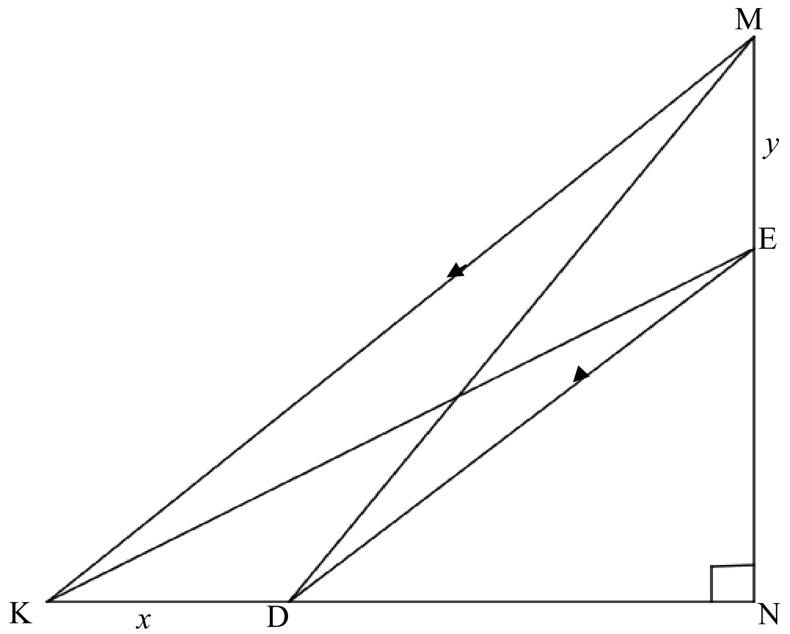
QUESTION 10

10.1 ΔABC is drawn such that MN is parallel to BC.



Prove that $\frac{AM}{MB} = \frac{AN}{NC}$ (5)

10.2 In ΔMNK; $\hat{N} = 90^0$ and D is a point on KN and E is a point on MN such that $DE \parallel KM$.
$ND : DK = 2 : 1$, $ME = y$ and $KD = x$.



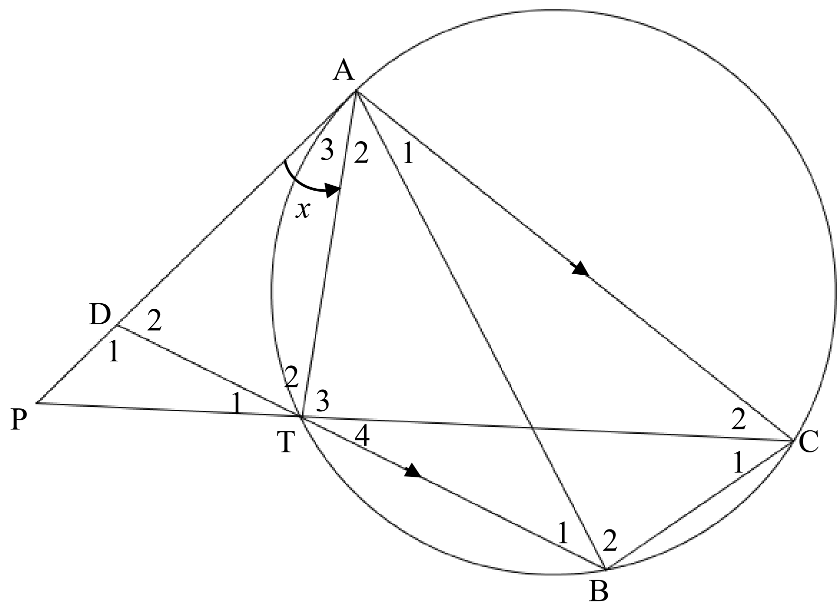
10.2.1 Determine the length of KM^2 in terms of x and y. (4)

10.2.2 Show that $\dfrac{DM^2 + KE^2}{KM^2} = \dfrac{13}{9}$ (4)

[13]

QUESTION 11

In the diagram, PA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are joined. $AC \parallel BD$.



11.1 Prove that $\Delta ABC \,|||\, \Delta ADT$. (6)

11.2 Prove that PT is a tangent to the circle ADT at T. (4)

11.3 Prove that $\Delta APT \,|||\, \Delta TPD$ (3)

11.4 If $AD = \frac{2}{3}AP$, show that $AP^2 = 3PT^2$. (4)

[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

InΔABC: $\quad \frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$