



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE/GRAAD 12**

**MATHEMATICS P2/WISKUNDE V2**  
**FEBRUARY/MARCH/FEBRUARIE/MAART 2016**  
**MEMORANDUM**

**MARKS: 150**  
**PUNTE: 150**

**This memorandum consists of 21 pages./**  
***Hierdie memorandum bestaan uit 21 bladsye.***

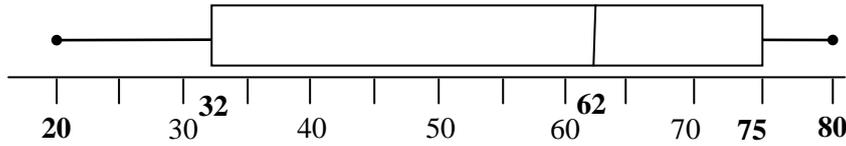
**NOTE:**

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**LET WEL:**

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

**QUESTION/VRAAG 1**

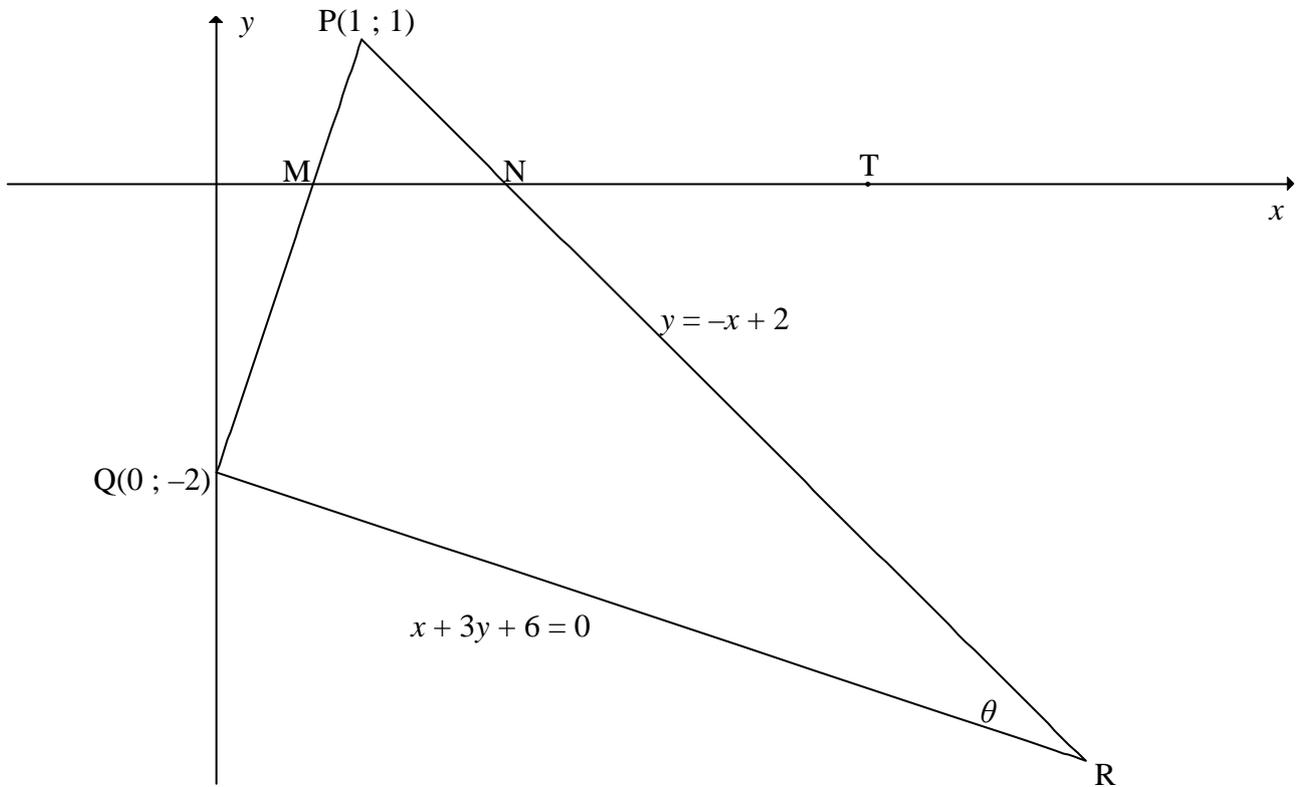


1.1	The data is skewed to the left/ <i>Die data is skeef na links.</i> <b>OR/OF</b> The data is negatively skewed/ <i>Die data is negatief skeef.</i>	✓ answ/antw ✓ answ/antw (1)									
1.2	Range/ <i>Omvang</i> = $80 - 20$ = 60	✓ max. – min. ✓ answ/antw (2)									
1.3	25% of the learners failed/ <i>van die leerdere het gedruip</i>	✓ ✓ answ/antw (2)									
1.4	$54 = \frac{445 + T_4}{9}$ $T_4 = 41$ <table border="1" data-bbox="336 1120 1082 1196" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">20</td> <td style="padding: 5px;">28</td> <td style="padding: 5px;">36</td> <td style="padding: 5px;">41</td> <td style="padding: 5px;">62</td> <td style="padding: 5px;">69</td> <td style="padding: 5px;">75</td> <td style="padding: 5px;">75</td> <td style="padding: 5px;">80</td> </tr> </table>	20	28	36	41	62	69	75	75	80	✓ 20 ✓✓ 41 ✓ 62 ✓ 75 ✓ 80 (6) <b>[11]</b>
20	28	36	41	62	69	75	75	80			

**QUESTION/VRAAG 2**

<p>2.1</p>	$\text{Mean/Gemiddelde} = \frac{2(15) + 8(25) + \dots + 2(85)}{60} = \frac{3080}{60}$ $= 51,33 \text{ messages per day/boodskappe per dag}$	<p>✓ 3 080                  ✓ <math>\frac{3080}{60}</math>                  ✓ answ/antw                  (3)</p>
<p>2.2</p>	<p style="text-align: center;"><b>OGIVE/OGIEF</b></p>	<p>✓ grounding at (10 ; 0)                  ✓ plotting at upper limits                  ✓ plotting. cumulative <i>f</i>                  ✓ smooth shape of curve                    ✓ geanker by (10 ; 0)                  ✓ stip by boonste limiete                  ✓ plot kumulatiewe <i>f</i>                  ✓ gladde vorm van kurwe                  (4)</p>
<p>2.3</p>	<p>Number of days/<i>Getal dae</i> = 60 – 46 (see on graph above/<i>sien op grafiek hierbo</i>)                  = 14 days/<i>dae</i></p> <p style="text-align: center;"><b>OR/OF</b></p> <p>Number of days/<i>Getal dae</i> = 2 + 3 + <math>\frac{1}{2} \times 18 = 14</math> days/<i>dae</i></p>	<p>✓ 46                  (accept 45 – 49)                  ✓ answ/antw                  (accept 11 – 15)                  (2)</p> <p>✓ add correct values/<i>tel korrekte waardes by</i>                  ✓ answ/antw                  (2)</p> <p style="text-align: right;"><b>[9]</b></p>

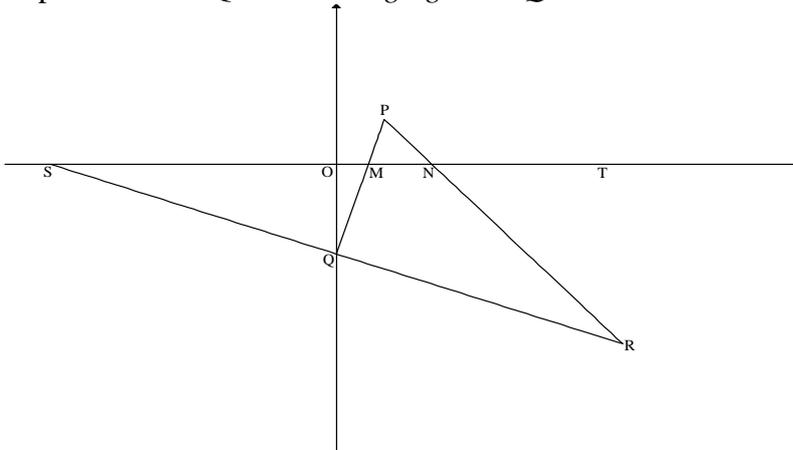
QUESTION/VRAAG 3



3.1	$m_{PQ} = \frac{1 - (-2)}{1 - 0}$ $= 3$	✓ subst (1 ; 1) & (0 ; -2) ✓ answ/antw (2)
3.2	$QR: y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore \hat{PQR} = 90^\circ$	$\checkmark m_{QR} = -\frac{1}{3}$ $\checkmark m_{PQ} \times m_{QR} = -1$ (2)

3.3	$-\frac{1}{3}x - 2 = -x + 2$ $\frac{2}{3}x = 4$ $x = 6$ $y = -4$ $\therefore R(6; -4)$	✓ equating/gelyk stel  ✓ x-value/waarde ✓ y-value/waarde (3)
3.4	$PR = \sqrt{(1-6)^2 + (1-(-4))^2}$ $= \sqrt{50} = 5\sqrt{2}$ <p style="text-align: center;"><b>OR/OF</b></p> $PR^2 = (1-6)^2 + (1-(-4))^2$ $= 50$ $\therefore PR = \sqrt{50} = 5\sqrt{2}$	✓ subst into/in distance formula/afstandsformule ✓ answ/antw in surd form/wortelvorm (2)  ✓ subst into/in distance formula/afstandsformule ✓ answ/antw in surd form/wortelvorm (2)
3.5	PR is a diameter/'n middellyn [chord subtends/kd onderspan 90°] Centre of circle/Midpt v sirkel: $\left(\frac{1+6}{2}; \frac{1-4}{2}\right)$ $= \left(3\frac{1}{2}; -1\frac{1}{2}\right)$ $r = \frac{\sqrt{50}}{2} \text{ OR } \frac{5\sqrt{2}}{2} \text{ OR } 3,54$ $\therefore \left(x - \frac{7}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{50}{4} \text{ OR } \frac{25}{2} \text{ OR } 12,5$	✓✓ S  ✓✓ $\left(3\frac{1}{2}; -1\frac{1}{2}\right)$  ✓ r-value/waarde  ✓ answ/antw (6)
3.6	$m$ of/van radius = -1 $\therefore m$ of/van tangent/raaklyn = 1 Equation of tangent/Vgl van raaklyn: $y - y_1 = (x - x_1)$ <span style="margin-left: 150px;"><math>y = x + c</math></span> $y - 1 = x - 1$ <span style="margin-left: 150px;"><b>OR/OF</b> <math>1 = 1 + c</math></span> $\therefore y = x$ <span style="margin-left: 150px;"><math>y = x</math></span>	✓ $m$ of tang/rkl  ✓ subst $m$ & P(1 ; 1) into/in eq of line/vgl v lyn ✓ answ/antw (3)
3.7	$\tan \hat{PNT} = m_{PR} = -1$ $\therefore \hat{PNT} = 135^\circ$ $\tan \hat{PMT} = m_{PQ} = 3$ $\therefore \hat{PMT} = 71,57^\circ$ $\hat{P} = 63,43^\circ$ [ext $\angle$ of $\Delta$ /buite $\angle$ v $\Delta$ ] $\therefore \theta = 26,57^\circ$ [sum of $\angle$ s in $\Delta$ /som v $\angle$ e in $\Delta$ ]  <p style="text-align: center;"><b>OR/OF</b></p>	✓ $\tan \hat{PNT} = -1$ ✓ $\hat{PNT} = 135^\circ$  ✓ $\hat{PMT} = 71,57^\circ$ ✓ $\hat{P} = 63,43^\circ$ ✓ answ/antw (5)

Extrapolation of RQ to S/Verlenging van RQ na S:



$$\tan \widehat{PNT} = m_{PR} = -1$$

$$\therefore \widehat{SNR} = 135^\circ$$

$$\tan \widehat{NSR} = m_{RS} = -\frac{1}{3}$$

$$\therefore \widehat{NSR} = 18,43^\circ$$

$$\theta = 180^\circ - (135^\circ + 18,43^\circ) \quad [\text{sum of } \angle\text{s in } \Delta/\text{som v } \angle\text{e in } \Delta]$$

$$= 26,57^\circ$$

**OR/OF**

$$PQ^2 = 1^2 + 3^2 = 10$$

$$PQ = \sqrt{10}$$

$$\therefore \sin \theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$$

$$\therefore \theta = 26,57^\circ$$

**OR/OF**

$$QR^2 = 6^2 + 2^2 = 40$$

$$QR = 2\sqrt{10}$$

$$\therefore \cos \theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$$

$$\therefore \theta = 26,57^\circ$$

**OR/OF**

$$\checkmark \tan \widehat{PNT} = -1$$

$$\checkmark \widehat{SNR} = 135^\circ$$

$$\checkmark \tan \widehat{NSR} = -\frac{1}{3}$$

$$\checkmark \widehat{NSR} = 18,43^\circ$$

$\checkmark$  answ/antw

(5)

$\checkmark$  subst into/in

distance formula/  
afstandsformule

$\checkmark$  distance/afst PQ

$\checkmark$  correct trig ratio/  
korrekte trig vhl

$\checkmark$  correct trig eq/  
korrekte trig vgl

$\checkmark$  answ/antw

(5)

$\checkmark$  subst into/in

distance formula/  
afstandsformule

$\checkmark$  distance/afst PQ

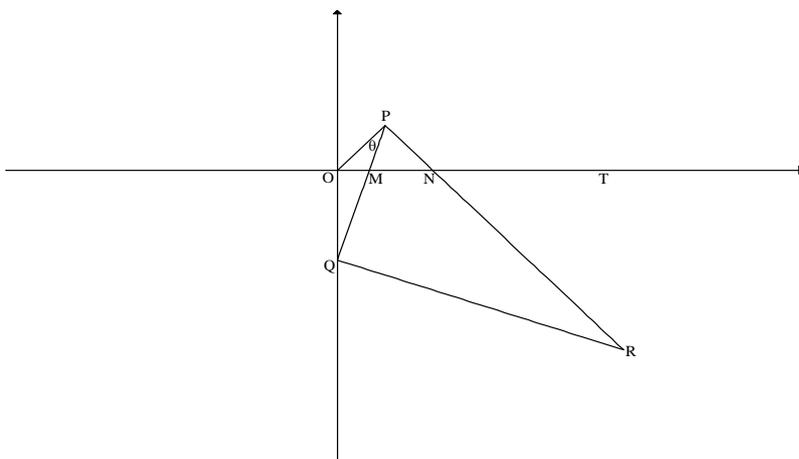
$\checkmark$  correct trig ratio/  
korrekte trig vhl

$\checkmark$  correct trig eq/  
korrekte trig vgl

$\checkmark$  answ/antw

(5)

$$\begin{aligned} \tan \theta &= \frac{m_{RQ} - m_{PR}}{1 + m_{RQ} \cdot m_{PR}} \\ &= \frac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)} \\ &= \frac{1}{2} \\ \therefore \theta &= 26,57^\circ \end{aligned}$$



tangent OP goes through the origin/raakl OP gaan deur oorsprong  
 $\widehat{POM} = 45^\circ$   
 $\widehat{OPM} = \theta = \widehat{P}$  [tan-chord theorem/raakl-kdst]  
 $\tan \widehat{PMT} = m_{PQ} = 3$   
 $\therefore \widehat{PMT} = 71,57^\circ$   
 $\therefore \theta + 45^\circ = 71,57^\circ$  [ext  $\angle$  of  $\Delta$ /buite- $\angle$  v  $\Delta$ ]  
 $\therefore \theta = 26,57^\circ$

✓ correct formula/  
 korrekte formule

✓  $m_{RQ} = -\frac{1}{3}$

✓ correct subst/  
 subst korrek

✓  $\tan \theta = \frac{1}{2}$

✓  $\theta = 26,57^\circ$

(5)

✓  $\widehat{POM} = 45^\circ$   
 ✓ R

✓  $\widehat{PMT} = 71,57^\circ$

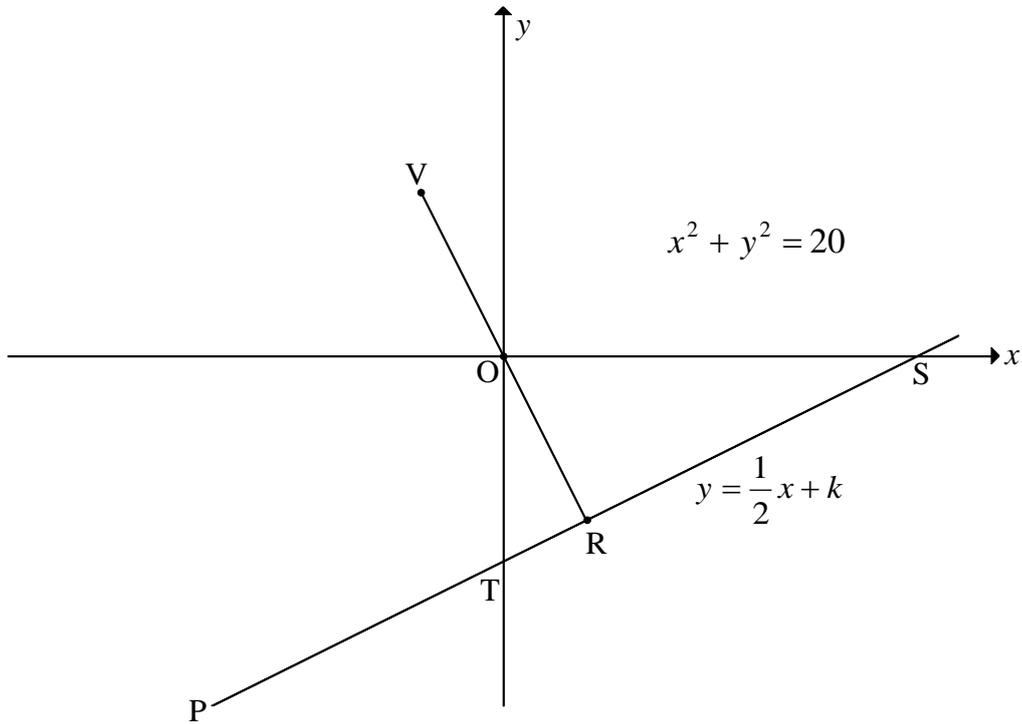
✓ S

✓  $\theta = 26,57^\circ$

(5)

[23]

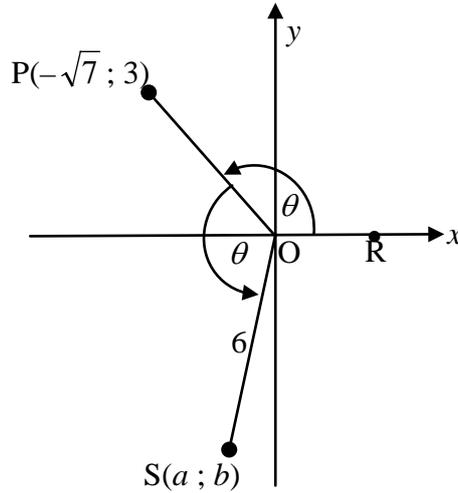
**QUESTION/VRAAG 4**



4.1	<p>OR <math>\perp</math> TR [radius <math>\perp</math> tangent/raakl]</p> <p><math>\therefore m_{TR} \times m_{OR} = -1</math></p> <p><math>\therefore m_{OR} = -2</math></p> <p><math>\therefore y = -2x</math></p>	<p>✓S/R</p> <p>✓m of/van OR</p> <p>✓equation/vgl (3)</p>
4.2	<p><math>x^2 + (-2x)^2 = 20</math></p> <p><math>x^2 + 4x^2 = 20</math></p> <p><math>5x^2 - 20 = 0</math></p> <p><math>x^2 - 4 = 0</math></p> <p><math>(x + 2)(x - 2) = 0</math></p> <p><math>\therefore x = 2</math></p> <p><math>y = -2(2) = -4</math></p> <p><math>\therefore R(2 ; -4)</math></p>	<p>✓subst eq of OR into circle eq/ subst vgl OR in sirkelvgl</p> <p>✓st. form/st. vorm</p> <p>✓x-value/waarde</p> <p>✓y-value/waarde (4)</p>

<p>4.3</p>	<p>Subst R(2 ; -4) into the equation of/in vgl van PRS:</p> $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore OT = 5$ $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore OS = 10$ $\text{Area/Oppervlakte} = \frac{1}{2} OS \cdot OT$ $= \frac{1}{2}(10)(5)$ $= 25 \text{ sq units/vk eenh}$	<p>✓ correct subst/ korrekte subst</p> <p>✓ value of k</p> <p>✓ y = 0 ✓ x-intercept/afsnit</p> <p>✓ correct subst into area form/ subst korrek in opp-formule</p> <p>✓ answ/antw</p> <p>(6)</p>
<p>4.4</p>	$0 = \frac{x_v + 2}{2} \quad \text{and/en} \quad 0 = \frac{y_v - 4}{2}$ $\therefore V(-2 ; 4)$ <p>T(0 ; -5) .... from/van 4.3</p> $VT = \sqrt{(-2 - 0)^2 + (4 - (-5))^2}$ $= \sqrt{4 + 81}$ $= \sqrt{85}$	<p>✓ x-value/waardeV ✓ y-value/waardeV</p> <p>✓ subst of points V and T into distance formula/ subst punte V en T in afst-form</p> <p>✓ answ/antw</p> <p>(4) [17]</p>

**QUESTION/VRAAG 5**



5.1.1	$\tan \theta = -\frac{3}{\sqrt{7}}$	✓ answ/antw (1)
5.1.2	$\sin(-\theta) = -\sin \theta$ $OP^2 = (-\sqrt{7})^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓ reduction/ reduksie  ✓ OP = 4  ✓ answ/antw (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2 \theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ <p style="text-align: center;"><b>OR/OF</b></p> $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2 \theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ <p style="text-align: center;"><b>OR/OF</b></p>	✓ trig ratio/verh  ✓ expansion/ uitbreiding  ✓ $\sin \theta = \frac{3}{4}$  ✓ answ/antw (4)  ✓ trig ratio/verh  ✓ expansion/ uitbreiding  ✓ $\cos \theta = \frac{-\sqrt{7}}{4}$  ✓ answ/antw (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2 \theta - \sin^2 \theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	<p>✓ trig ratio/verh</p> <p>✓ expansion/ uitbreiding</p> <p>✓ <math>\cos \theta = \frac{-\sqrt{7}}{4}</math> &amp;</p> <p><math>\sin \theta = \frac{3}{4}</math></p> <p>✓ answ/antw</p> <p>(4)</p>
5.2.1	$\frac{4 \sin x \cdot \cos x}{2 \sin^2 x - 1} = \frac{2(2 \sin x \cdot \cos x)}{-(1 - 2 \sin^2 x)}$ $= \frac{2 \sin 2x}{-\cos 2x}$ $= -2 \tan 2x$	<p>✓ <math>2 \sin 2x</math></p> <p>✓ <math>-\cos 2x</math></p> <p>✓ answ/antw</p> <p>(3)</p>
5.2.2	$\frac{4 \sin 15^\circ \cos 15^\circ}{2 \sin^2 15^\circ - 1} = -2 \tan 2(15^\circ)$ $= -2 \tan 30^\circ$ $= -2 \left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}} \text{ OR/OR } -\frac{2\sqrt{3}}{3}$	<p>✓ <math>-2 \tan 2(15^\circ)</math></p> <p>✓ answ/antw</p> <p>(2)</p> <p><b>[13]</b></p>

**QUESTION/VRAAG 6**

<p>6.1</p>	$\sin(x + 60^\circ) + 2\cos x = 0$ $\sin x \cos 60^\circ + \cos x \sin 60^\circ + 2\cos x = 0$ $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$ $\frac{1}{2}\sin x = -2\cos x - \frac{\sqrt{3}}{2}\cos x$ $\sin x = -4\cos x - \sqrt{3}\cos x$ $\sin x = \cos x(-4 - \sqrt{3})$ $\frac{\sin x}{\cos x} = \frac{\cos x(-4 - \sqrt{3})}{\cos x}$ $\therefore \tan x = -4 - \sqrt{3}$	<p>✓ expansion/uitbreiding</p> <p>✓ special angle values/ spesiale <math>\angle</math>-waardes</p> <p>✓ simpl/vereenv</p> <p>✓</p> <p><math>\sin x = \cos x(-4 - \sqrt{3})</math></p> <p>(4)</p>
<p>6.2</p>	$\tan x = -4 - \sqrt{3}$ $\tan x = -(4 + \sqrt{3})$ <p>ref <math>\angle = 80,10^\circ</math></p> <p><math>x = -80,1^\circ</math> or/of <math>99,9^\circ</math></p>	<p>✓ <math>80,10^\circ</math></p> <p>✓ <math>99,90^\circ</math></p> <p>✓ <math>-80,1^\circ</math></p> <p>(3)</p>
<p>6.3.1</p>		<p>✓ <math>(30^\circ ; 1)</math></p> <p>✓ <math>(-60^\circ ; 0)</math></p> <p>✓ shape/vorm</p> <p>(3)</p>
<p>6.3.2</p>	$\therefore \sin(x + 60^\circ) > -2\cos x$ $x \in (-80,10^\circ ; 99,90^\circ) \text{ OR/OF } -80,10^\circ < x < 99,90^\circ$	<p>✓ ✓ critical values/ kritiese waardes</p> <p>✓ notation/notasie</p> <p>(3)</p> <p><b>[13]</b></p>

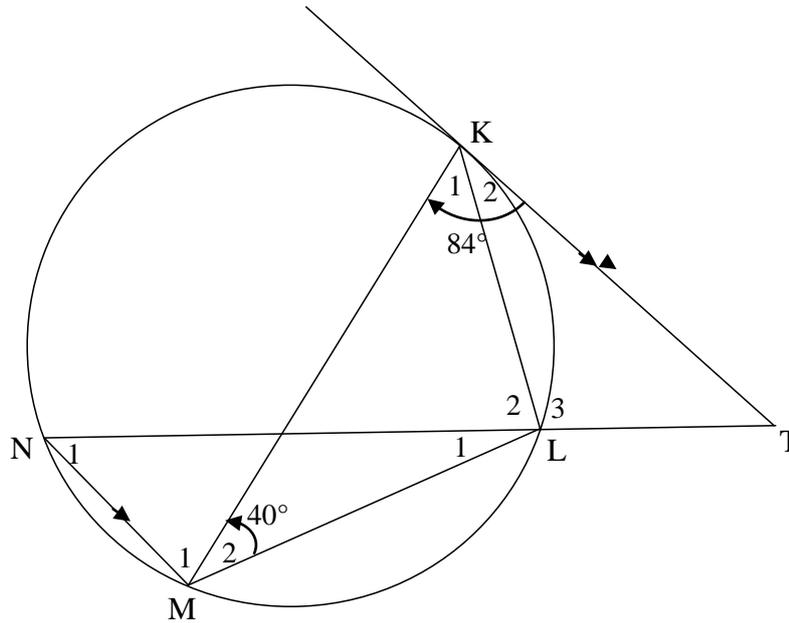
**QUESTION/VRAAG 7**

7.1.1	$\begin{aligned} \text{Area of/Oppervlakte van } \Delta PQR &= \frac{1}{2} PQ \cdot QR \cdot \sin \hat{Q} \\ &= \frac{1}{2} x(20 - 4x)(\sin 60^\circ) \\ &= 10x - 2x^2 \left( \frac{\sqrt{3}}{2} \right) \\ &= 5\sqrt{3}x - \sqrt{3}x^2 \end{aligned}$	<p>✓ subst into area rule/ subst in opp-reël ✓ subst &amp; simpl/ subst en vereenv (2)</p>
7.1.2	<p>For maximum area/Vir maksimum opp:  <math>(\text{Area } \Delta PQR)' = 0</math>  <math>5\sqrt{3} - 2\sqrt{3}x = 0</math>  <math>2\sqrt{3}x = 5\sqrt{3}</math>  <math>\therefore x_{\max} = \frac{5}{2}</math> or <math>2\frac{1}{2}</math> or/of 2,5</p> <p><b>OR/OF</b></p> $x_{\max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2} \text{ or } 2\frac{1}{2} \text{ or } 2,5$ <p><b>OR/OF</b></p> $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5 - x) = 0$ $\therefore x = 0 \text{ or } 5$ $\therefore x_{\max} = \frac{0+5}{2} = \frac{5}{2} \text{ or/of } 2,5$	<p>✓ (Area <math>\Delta PQR</math>)' = 0  <math>5\sqrt{3} - 2\sqrt{3}x</math>  <math>2\sqrt{3}x = 5\sqrt{3}</math>  <math>\therefore x_{\max} = \frac{5}{2}</math> or <math>2\frac{1}{2}</math> or/of 2,5          ✓ answ/antw          (3)</p> <p>✓ formula/e          ✓ subst          ✓ answ/antw          (3)</p> <p>✓ x-intercepts/          x-afsnitte          ✓ subst          ✓ answ/antw          (3)</p>
7.1.3	$\begin{aligned} RP^2 &= QP^2 + QR^2 - 2 \cdot QP \cdot QR \cdot \cos Q \\ &= 10^2 + 2,5^2 - 2(10)(2,5) \cos 60^\circ \\ &= 81,25 \\ \therefore RP &= 9,01 \end{aligned}$	<p>✓ subst into cosine rule/in cos-reël          ✓ simpl/vereenv          ✓ answ/antw          (3)</p>

<p>7.2</p>	<p>In <math>\triangle ABC</math>: <math>\sin \beta = \frac{h}{AB}</math>  <math>\therefore AB = \frac{h}{\sin \beta}</math></p> <p>In <math>\triangle ABD</math>: <math>AB = BD</math> and/en <math>\hat{A}DB = 90^\circ - \beta</math> [<math>\angle</math>s of/v <math>\triangle = 180^\circ</math>]  <math>\frac{\sin 2\beta}{AD} = \frac{\sin(90^\circ - \beta)}{AB}</math>  <math>AD = \frac{AB \cdot \sin 2\beta}{\sin(90^\circ - \beta)}</math>  <math>= \frac{h}{\sin \beta} \times \frac{2 \sin \beta \cdot \cos \beta}{\cos \beta}</math>  <math>= 2h</math></p> <p><b>OR/OF</b></p> <p>In <math>\triangle ABC</math>: <math>\sin \beta = \frac{h}{AB}</math>  <math>\therefore AB = \frac{h}{\sin \beta}</math></p> <p>In <math>\triangle ABD</math>: <math>AB = BD</math>  <math>AD^2 = AB^2 + AB^2 - 2AB \cdot AB \cdot \cos 2\beta</math>  <math>= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 \cdot \cos 2\beta</math>  <math>= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 (1 - 2 \sin^2 \beta)</math>  <math>= \left(\frac{h}{\sin \beta}\right)^2 + \left(\frac{h}{\sin \beta}\right)^2 - 2\left(\frac{h}{\sin \beta}\right)^2 + 4h^2</math>  <math>= 4h^2</math>  <math>\therefore AD = 2h</math></p> <p><b>OR/OF</b></p> <p>Split isosceles triangle <math>ABQ</math> into two congruent triangles <math>AEB</math> and <math>DEB</math>. Then <math>\triangle ABC \equiv \triangle BAE</math> (<math>AB = AC</math>, <math>\hat{A}BE = \hat{A}CB = \beta</math>, <math>h</math>)  <math>\therefore AE = ED = BC = h</math>  <math>\therefore AD = 2h</math></p>	<p>✓ AB ito <math>h</math> and/en <math>\beta</math></p> <p>✓ <math>\hat{A}DB = 90^\circ - \beta</math></p> <p>✓ correct subst into cosine rule/subst          korrek in cos-reël</p> <p>✓ AD as subject/          onderwerp</p> <p>✓ expansion/uitbrei</p> <p>✓ <math>\sin(90^\circ - \beta)</math>  <math>= \cos \beta</math></p> <p>✓ answer ito <math>h</math></p> <p>(7)</p> <p>✓ AB ito <math>h</math> and/en <math>\beta</math></p> <p>✓ correct subst into cosine rule/subst          korrek in cos-reël</p> <p>✓ expansion/uitbrei</p> <p>✓ multiplication/          vermenigv</p> <p>✓ simpl/vereenv</p> <p>✓ answer ito <math>h</math></p> <p>(7)</p> <p>(7)</p>
		<b>[15]</b>

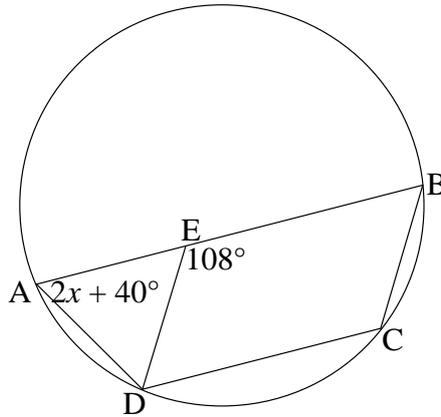
QUESTION/VRAAG 8

8.1



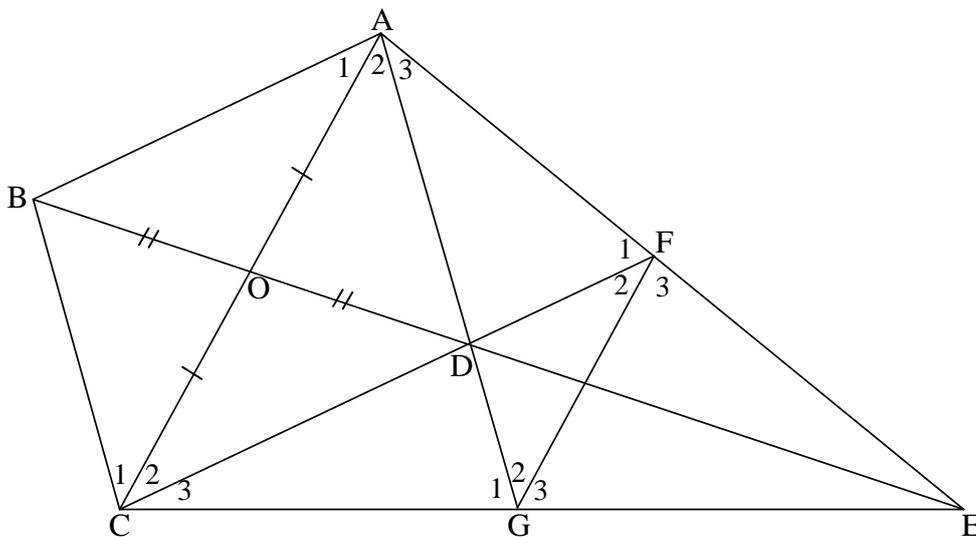
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40^\circ$ [tan chord theorem/raakl-kdst]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [ $\angle$ s in the same seg/ $\angle$ e in dies segm] $\hat{K}_1 = 84^\circ - 40^\circ = 44^\circ$ $\therefore \hat{N}_1 = 44^\circ$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44^\circ$ [alt/verw $\angle$ s/e; $KT \parallel NM$ ]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ $= 40^\circ + 44^\circ$ $= 84^\circ$	✓R ✓S (2)
8.1.5	In $\Delta KLM$ : $44^\circ + 84^\circ + 40^\circ + \hat{L}_1 = 180^\circ$ [ $\angle$ s sum in $\Delta$ / $\angle$ e som in $\Delta$ ] $\therefore \hat{L}_1 = 12^\circ$	✓S (1)

8.2



8.2	$\hat{C} = 108^\circ$ $2x + 40^\circ + 108^\circ = 180^\circ$ $2x = 32^\circ$ $x = 16^\circ$  <p style="text-align: center;"><b>OR/OF</b></p> $\hat{C} = 180^\circ - (2x + 40^\circ)$ $180^\circ - (2x + 40^\circ) = 108^\circ$ $2x = 32^\circ$ $x = 16^\circ$	<p>[opp<math>\angle</math>s of <math>\parallel</math>m/tos <math>\angle</math>e v <math>\parallel</math>m]                  [opp<math>\angle</math>s of cyc quad/tos <math>\angle</math>e v <math>kdvh</math>]</p> <p><math>\checkmark</math>S <math>\checkmark</math>R  <math>\checkmark</math>S <math>\checkmark</math>R</p> <p><math>\checkmark</math>answ/antw</p> <p style="text-align: right;">(5)</p> <p>[opp<math>\angle</math>s of cyc quad/tos <math>\angle</math>e v <math>kdvh</math>]                  [opp<math>\angle</math>s of <math>\parallel</math>m/tos <math>\angle</math>e v <math>\parallel</math>m]</p> <p><math>\checkmark</math>S <math>\checkmark</math>R  <math>\checkmark</math>S <math>\checkmark</math>R</p> <p><math>\checkmark</math>answ/antw</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;"><b>[15]</b></p>
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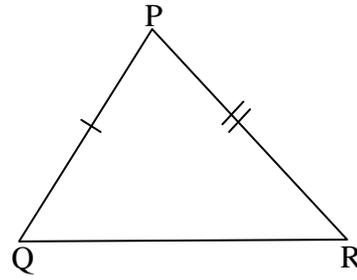
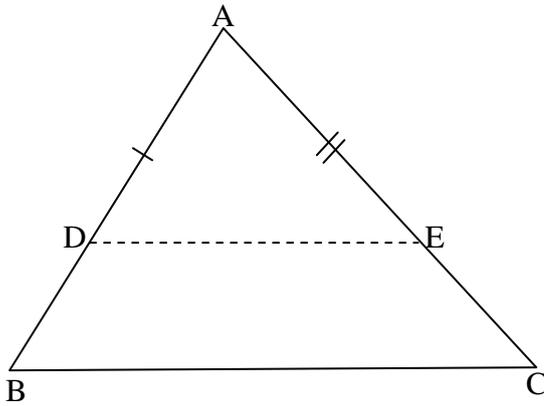
**QUESTION/VRAAG 9**



9.1	ABCD is a   m [diags of quad bisect each other/ <i>hoekl v vh halveer mekaar</i> ]	✓ R (1)
9.2	$\frac{ED}{DB} = \frac{FE}{AF}$ [Prop Th/Eweredigh st; DF    BA] $\frac{ED}{DB} = \frac{GE}{CG}$ [Prop Th/Eweredigh st; DG    BC]	✓ S ✓ R ✓ S ✓ R (4)
9.3	$\frac{FE}{AF} = \frac{GE}{CG}$ [proved/bewys] $\therefore AC \parallel FG$ [line divides two sides of $\Delta$ in prop/ <i>lyn verdeel 2 sye van <math>\Delta</math> eweredig</i> ] $\hat{C}_2 = \hat{F}_2$ [alt/verw $\angle$ s/e; AC    FG] $\hat{A}_1 = \hat{C}_2$ [alt/verw $\angle$ s/e; AB    CD] $\therefore \hat{A}_1 = \hat{F}_2$	✓ S ✓ S ✓ R ✓ S ✓ S (5)
9.4	$\hat{A}_1 = \hat{A}_2$ [diags of rhombus/ <i>hoekl v ruit</i> ] $\hat{A}_2 = \hat{F}_2$ [ $\hat{A}_1 = \hat{F}_2$ ] $\therefore ACGF = \text{cyc quad/kdvh}$ [ $\angle$ s in the same seg =/ <i><math>\angle</math>e in dies segm =</i> ]  <b>OR/OF</b>  $\hat{C}_2 = \hat{A}_2$ [ $\angle$ s opp equal sides of rhombus/ <i><math>\angle</math>e to gelyke sye v ruit</i> ] $\hat{A}_2 = \hat{G}_2$ [alt/verw- $\angle$ s/e; AC    FG] $\therefore \hat{C}_2 = \hat{G}_2$ $\therefore ACGF$ is a cyc quad/kdvh [ $\angle$ s in the same seg =/ <i><math>\angle</math>e in dies segm =</i> ]	✓ S ✓ S ✓ R  ✓ S ✓ S ✓ R (3)  (3) <b>[13]</b>

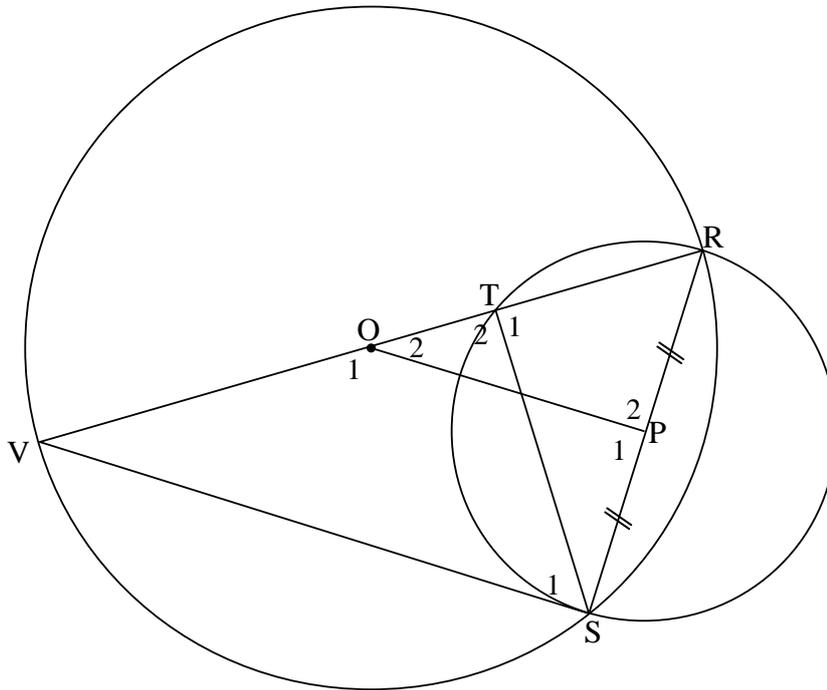
**QUESTION/VRAAG 10**

10.1



10.1.1	In $\triangle ADE$ and/en $\triangle PQR$ : $AD = PQ$ [construction/konstr] $\hat{A} = \hat{P}$ [given/gegee] $AE = PR$ [construction/konstr] $\therefore \triangle ADE \equiv \triangle PQR$ [S/S]	$\checkmark$ all/al 3 S's/e  $\checkmark$ reason/rede (2)
10.1.2	$\hat{A}DE = \hat{Q}$ [Δs $\equiv \therefore$ corres/ooreenk $\angle$ s/e =] But $\hat{B} = \hat{Q}$ [given/gegee] $\therefore \hat{A}DE = \hat{B}$ $\therefore DE \parallel BC$ [corres/ooreenk $\angle$ s/e =]	$\checkmark \hat{A}DE = \hat{Q}$  $\checkmark \hat{A}DE = \hat{B}$ $\checkmark$ reason/rede (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/Eweredigh st; $DE \parallel BC$ ] But/Maar $AD = PQ$ and/en $AE = PR$ [construction/konstr] $\therefore \frac{AB}{PQ} = \frac{AC}{PR}$	$\checkmark$ S/R $\checkmark$ S (2)

10.2



10.2.1	line from centre to midpt of chord/ <i>lyn van midpt na midpt van koord</i>	✓ answ/antw (1)
10.2.2	<p><math>OP \parallel VS</math> [Midpt Theorem/Midpt-stelling]</p> <p>In <math>\triangle ROP</math> and/en <math>\triangle RVS</math>:</p> <p><math>\hat{R} = \hat{R}</math> [common/gemeen]</p> <p><math>\hat{O}_2 = \hat{V}</math> [corresp/ooreenk <math>\angle</math>s/e; <math>OP \parallel VS</math>]</p> <p><math>\therefore \triangle ROP \parallel \triangle RVS</math> [<math>\angle, \angle, \angle</math>]</p> <p style="text-align: center;"><b>OR/OF</b></p> <p>In <math>\triangle ROP</math> and/en <math>\triangle RVS</math>:</p> <p><math>\hat{P}_2 = \hat{S}_R</math> [corresponding <math>\angle</math>s/ ooreenkomstige <math>\angle</math>'e]</p> <p><math>\hat{R} = \hat{R}</math> [common/gemeen]</p> <p><math>\therefore \triangle ROP \parallel \triangle RVS</math> [<math>\angle, \angle, \angle</math>]</p>	<p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S &amp; <math>\angle; \angle; \angle</math></p> <p><b>OR/OF</b></p> <p>3 angles/hoeke (4)</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S &amp; <math>\angle; \angle; \angle</math></p> <p><b>OR/OF</b></p> <p>3 angles/hoeke (4)</p>

10.2.3	<p>In <math>\triangle RVS</math> and/en <math>\triangle RST</math>:  <math>\widehat{VSR} = \widehat{STR} = 90^\circ</math> [<math>\angle</math> in semi-circle/<math>\angle</math> in halfsirkel]  <math>\widehat{R}</math> is common/gemeen  <math>\widehat{V} = \widehat{TSR}</math>  <math>\therefore \triangle RVS \parallel \triangle RST</math> [<math>\angle, \angle, \angle</math>]</p>	<p>✓S ✓R                  ✓S &amp; <math>\angle; \angle; \angle</math>  <b>OR/OF</b>                  3 angles/hoeke                  (3)</p>
10.2.4	<p>In <math>\triangle RTS</math> and/en <math>\triangle STV</math>:  <math>\widehat{RTS} = \widehat{VTS} = 90^\circ</math> [<math>\angle</math> s on straight line/<math>\angle</math> e op rt lyn]  <math>\widehat{R} = 90^\circ - \widehat{TSR}</math>  <math>= \widehat{TSV}</math>  <math>\widehat{TSR} = \widehat{V}</math>  <math>\therefore \triangle RTS \parallel \triangle STV</math> [<math>\angle, \angle, \angle</math>]  <math>\therefore \frac{RT}{ST} = \frac{TS}{VT}</math>  <math>\therefore ST^2 = VT \cdot TR</math></p>	<p>✓<math>\triangle RTS</math> &amp; <math>\triangle STV</math>                  ✓S                  ✓S                  ✓S (with justification/met motivering)                  ✓<math>\triangle RTS \parallel \triangle STV</math>                  ✓ratio/verh                  (6)</p>
		<b>[21]</b>

**TOTAL/TOTAAL: 150**