



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2017

MARKS: 150

TIME: 3 hours

**This question paper consists of 13 pages, 1 information sheet
and an answer book of 27 pages.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

An IT company writes programs for apps. The time taken (in hours) to write the programs and the cost (in thousands of rands) are shown in the table below.

TIME TAKEN (IN HOURS)	5	7	5	8	10	13	15	20	18	25	23
COST (IN THOUSANDS OF RANDS)	10	10	15	12	20	25	28	32	28	40	30

- 1.1 Determine the equation of the least squares regression line. (3)
- 1.2 Use the equation of the least squares regression line to predict the cost, in rands, of an app that will take 16 hours to write. (2)
- 1.3 Calculate the correlation coefficient of the data. (1)
- 1.4 For each app that the company writes, there is a cost that is independent of the number of hours spent on writing the app. Calculate this cost (in rands). (2)
- [8]**

QUESTION 2

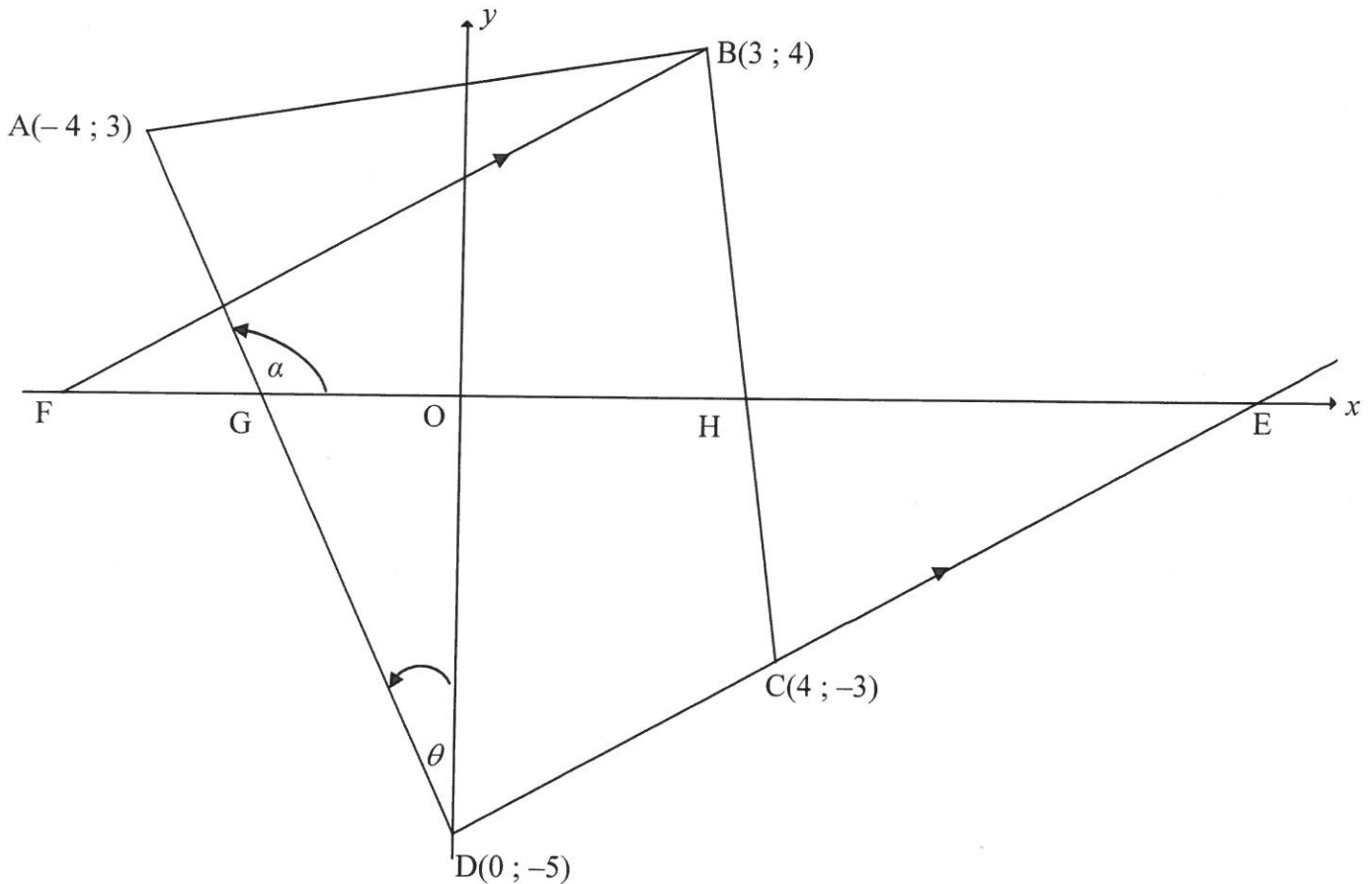
The commission earned, in thousands of rands, by the salesmen of a particular company in a certain month is shown in the table below.

COMMISSION EARNED (IN THOUSANDS OF RANDS)	FREQUENCY
$20 < x \leq 40$	7
$40 < x \leq 60$	6
$60 < x \leq 80$	8
$80 < x \leq 100$	10
$100 < x \leq 120$	4

- 2.1 Write down the modal class of the data. (1)
- 2.2 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)
- 2.3 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)
- 2.4 A salesman receives a bonus if his commission is more than R90 000 for the month. Calculate how many of the salesmen received bonuses for this month. (2)
- 2.5 Determine the approximate mean commission earned by the salesmen in this month correct to the nearest thousand rand. (3)
- [12]**

QUESTION 3

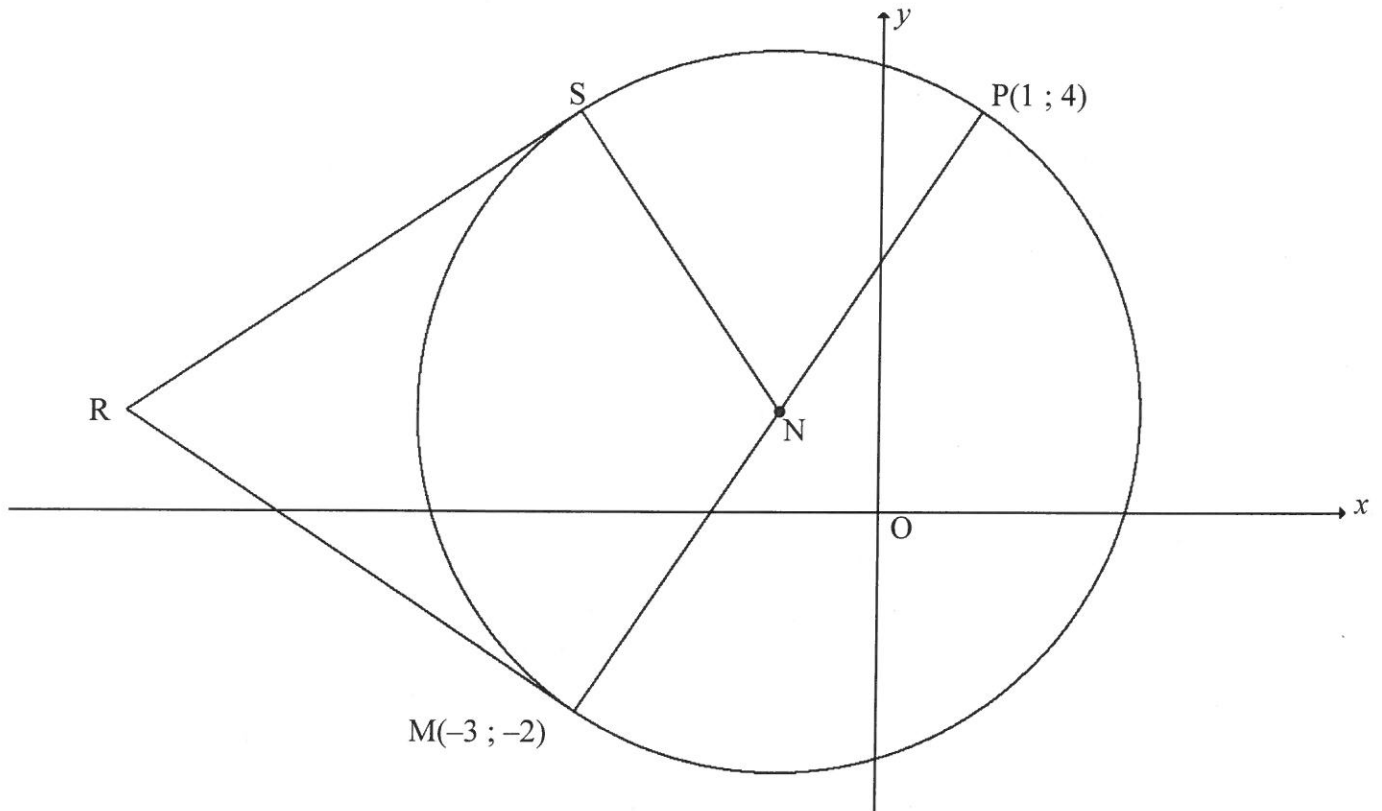
In the diagram, ABCD is a quadrilateral having vertices $A(-4 ; 3)$, $B(3 ; 4)$, $C(4 ; -3)$ and $D(0 ; -5)$. DC produced cuts the x -axis at E, BC cuts the x -axis at H and AD cuts the x -axis at G. F is a point on the x -axis such that $BF \parallel DE$. $\widehat{AGO} = \alpha$ and $\widehat{ADO} = \theta$.



- 3.1 Calculate the gradient of DC. (2)
 - 3.2 Prove that $AD \perp DC$. (3)
 - 3.3 Show by calculation that $\triangle ABC$ is an isosceles. (4)
 - 3.4 Determine the equation of BF in the form $y = mx + c$. (3)
 - 3.5 Calculate the size of θ . (3)
 - 3.6 Determine the equation of the circle, with the centre as the origin and passing through point C, in the form $x^2 + y^2 = r^2$. (2)
- [17]**

QUESTION 4

In the diagram, N is the centre of the circle. $M(-3 ; -2)$ and $P(1 ; 4)$ are points on the circle. MNP is the diameter of the circle. Tangents drawn to circle N from point R , outside the circle, meet the circle at S and M respectively.



- 4.1 Determine the coordinates of N . (3)
- 4.2 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (4)
- 4.3 Determine the equation of the tangent RM in the form $y = mx + c$. (5)
- 4.4 If it is given that the line joining S to M is perpendicular to the x -axis, determine the coordinates of S . (2)
- 4.5 Determine the coordinates of R , the common external point from which both tangents to the circle are drawn. (4)
- 4.6 Calculate the area of $RSNM$. (4)

[22]

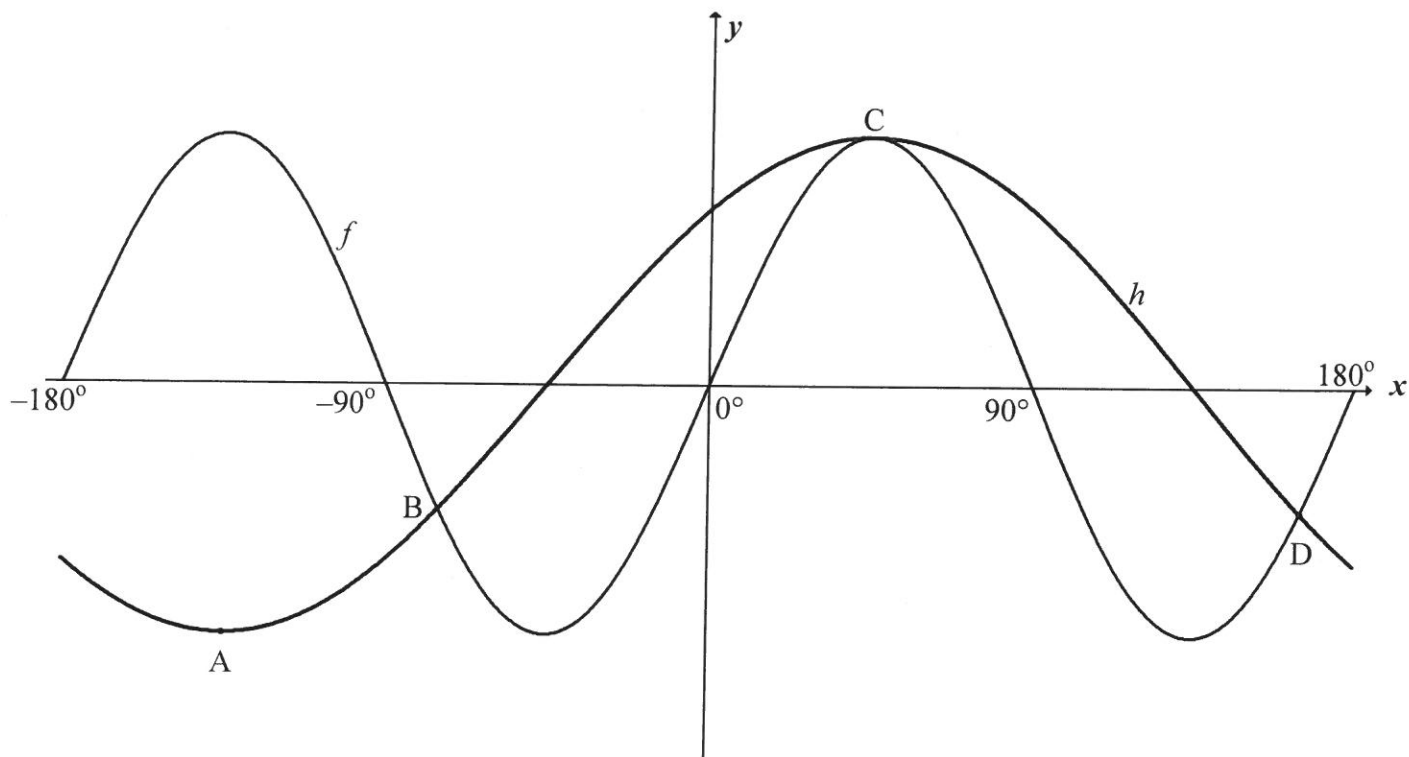
QUESTION 5

- 5.1 Given: $\sin A = 2p$ and $\cos A = p$
- 5.1.1 Determine the value of $\tan A$. (2)
- 5.1.2 **Without using a calculator**, determine the value of p , if $A \in [180^\circ ; 270^\circ]$. (3)
- 5.2 Determine the general solution of $2\sin^2 x - 5\sin x + 2 = 0$ (6)
- 5.3 5.3.1 Expand $\sin(x + 300^\circ)$ using an appropriate compound angle formula. (1)
- 5.3.2 **Without using a calculator**, determine the value of $\sin(x + 300^\circ) - \cos(x - 150^\circ)$. (5)
- 5.4 Prove the identity: $\frac{\tan x + 1}{\sin x \tan x + \cos x} = \sin x + \cos x$. (5)
- 5.5 Consider: $\sin x + \cos x = \sqrt{1 + k}$
- 5.5.1 Determine k as a single trigonometric ratio. (3)
- 5.5.2 Hence, determine the maximum value of $\sin x + \cos x$. (2)

[27]

QUESTION 6

In the diagram are the graphs of $f(x) = \sin 2x$ and $h(x) = \cos(x - 45^\circ)$ for the interval $x \in [-180^\circ ; 180^\circ]$. $A(-135^\circ ; -1)$ is a minimum point on graph h and $C(45^\circ ; 1)$ is a maximum point on both graphs. The two graphs intersect at B, C and $D\left(165^\circ ; -\frac{1}{2}\right)$.



- 6.1 Write down the period of f . (1)
 - 6.2 Determine the x -coordinate of B . (1)
 - 6.3 Use the graphs to solve $2 \sin x \cdot \cos x \leq \frac{1}{\sqrt{2}}(\cos x + \sin x)$ for the interval $x \in [-180^\circ ; 180^\circ]$. Show ALL working. (4)
- [6]**

QUESTION 7

A rectangular box with lid $ABCD$ is given in FIGURE (i) below. The lid is opened through 60° to position $HKCD$, as shown in the FIGURE (ii) below. $EF = 12$ cm, $FG = 6$ cm and $BG = 8$ cm.

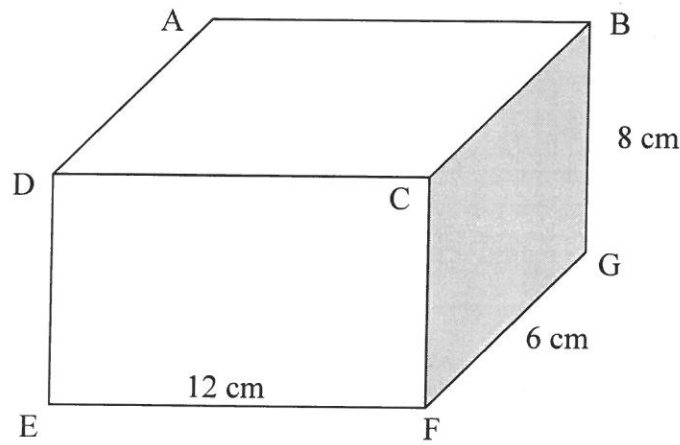


FIGURE (I)

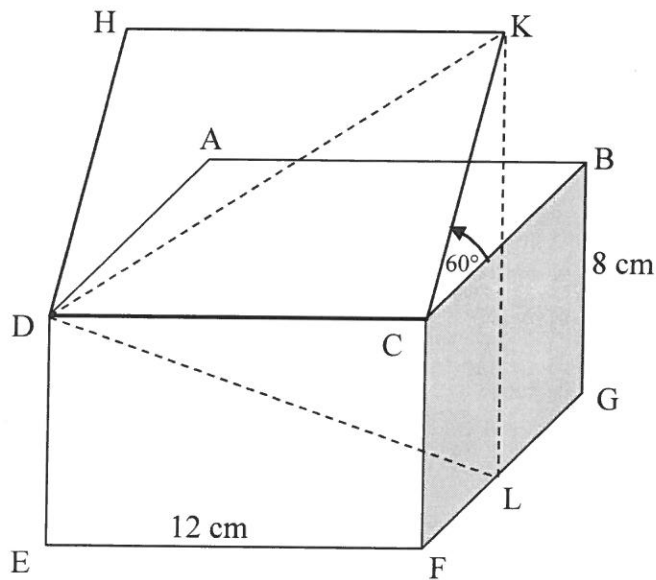


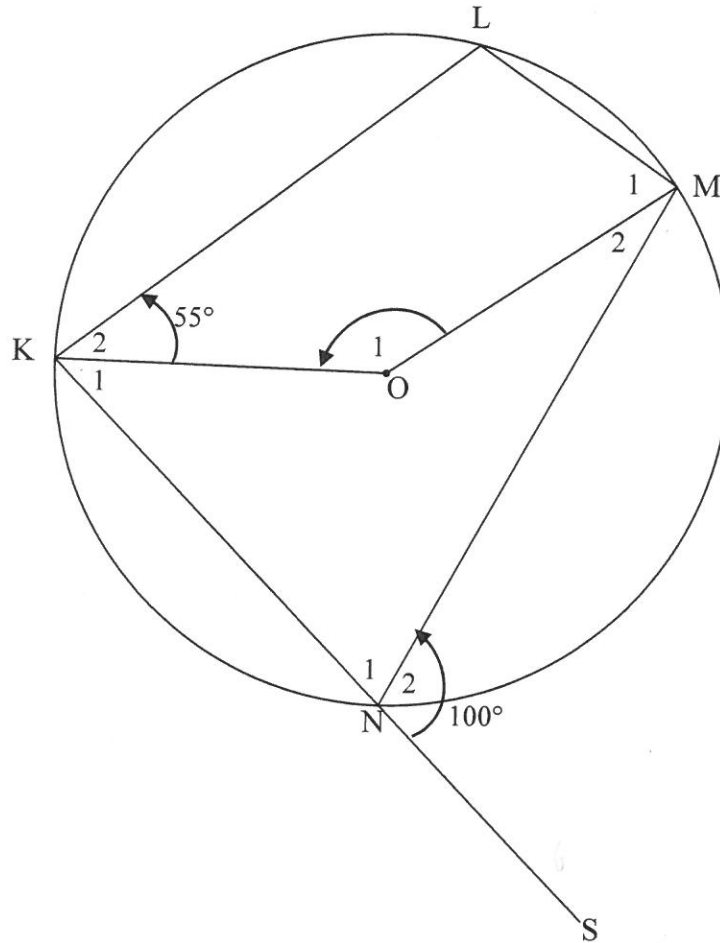
FIGURE (II)

- 7.1 Write down the length of KC . (1)
- 7.2 Determine KL , the perpendicular height of K , above the base of the box. (3)
- 7.3 Hence, determine the value of $\frac{\sin \hat{KDL}}{\sin \hat{DLK}}$. (4)

[8]

QUESTION 8

In the diagram, O is the centre of circle KLMN and KO and OM are joined. Chord KN is produced to S. $\hat{K}_2 = 55^\circ$ and $\hat{N}_2 = 100^\circ$.

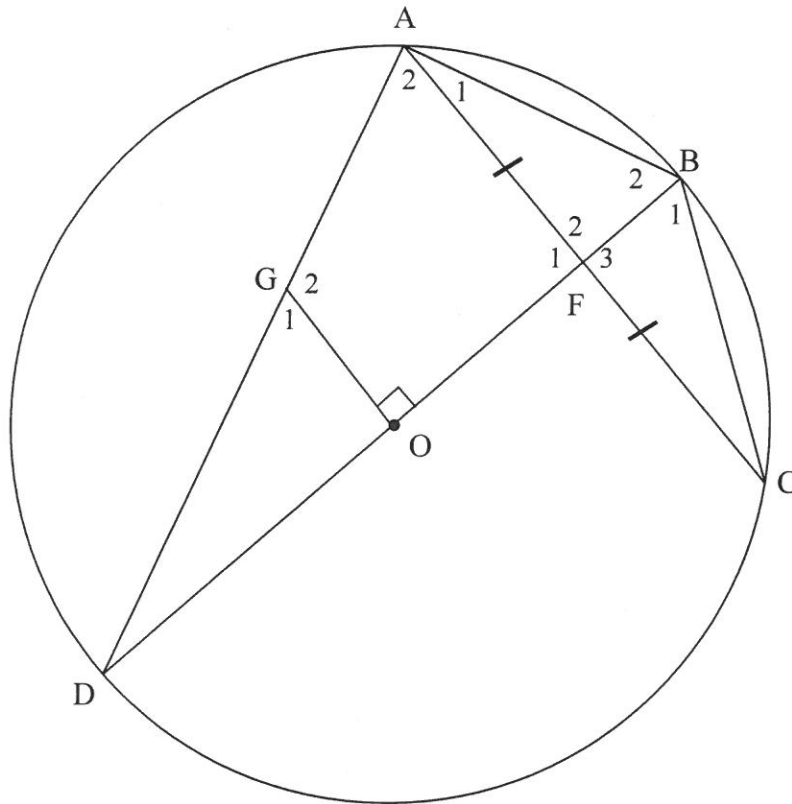


Determine, with reasons, the size of the following:

- 8.1 \hat{L} (2)
 - 8.2 \hat{O}_1 (3)
 - 8.3 \hat{M}_1 (2)
- [7]

QUESTION 9

In the diagram, O is the centre of circle ABCD and BOD is a diameter. F, the midpoint of chord AC, lies on BOD. G is a point on AD such that $GO \perp DB$.



9.1 Give a reason why:

9.1.1 $\hat{DAB} = 90^\circ$ (1)

9.1.2 AGOB is a cyclic quadrilateral (1)

9.2 Prove that:

9.2.1 $AC \parallel GO$ (3)

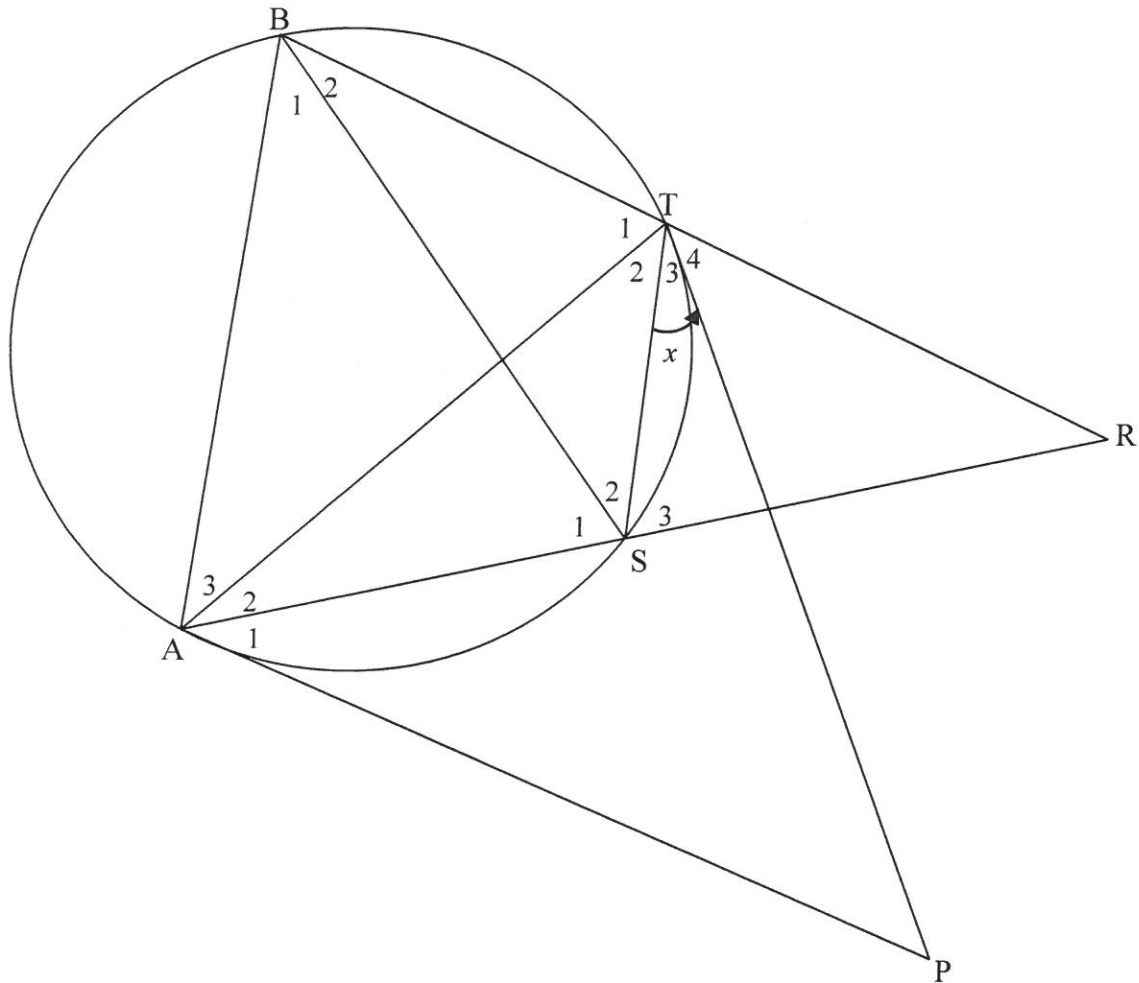
9.2.2 $\hat{G}_1 = \hat{B}_1$ (4)

9.3 If it is given that $FB = \frac{2}{5}r$, where r is the radius of the circle, determine, with reasons, the ratio of $\frac{DG}{DA}$.

(3)
[12]

QUESTION 10

In the diagram, PA and PT are tangents to a circle at A and T respectively. B and S are points on the circle such that BT produced and AS produced meet at R and $BR = AR$. BS, AT and TS are drawn. $\hat{T}_3 = x$.



10.1 Give a reason why $\hat{T}_3 = \hat{A}_2 = x$. (1)

10.2 Prove that:

10.2.1 $AB \parallel ST$ (5)

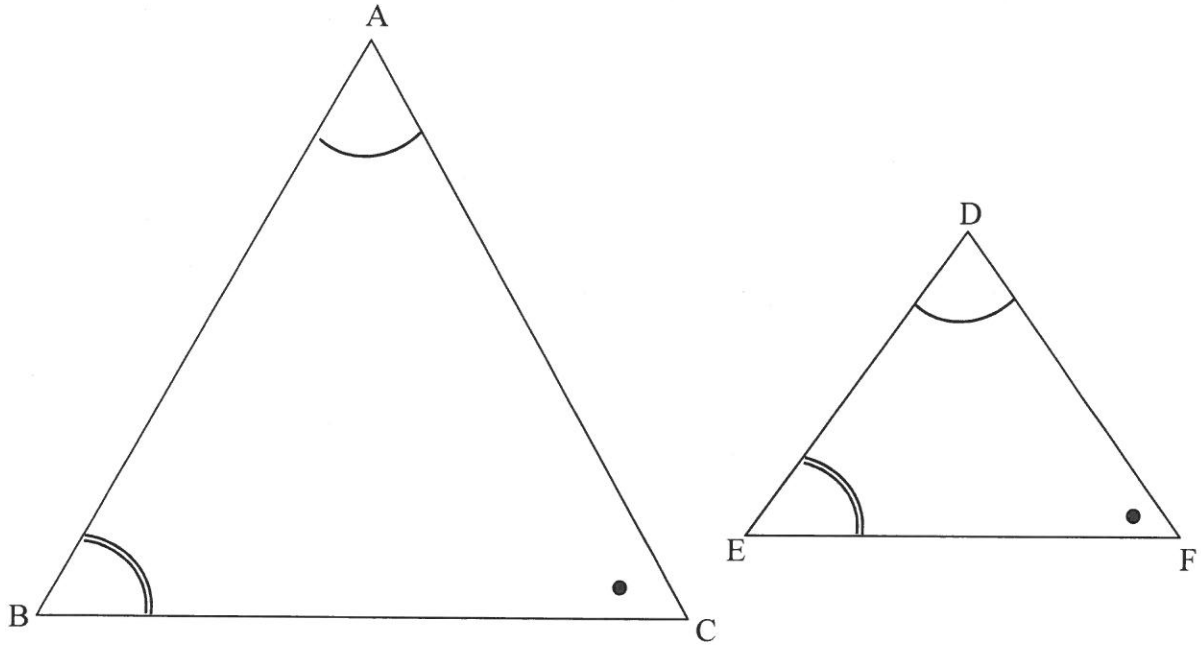
10.2.2 $\hat{T}_4 = \hat{A}_1$ (5)

10.2.3 RTAP is a cyclic quadrilateral (2)

[13]

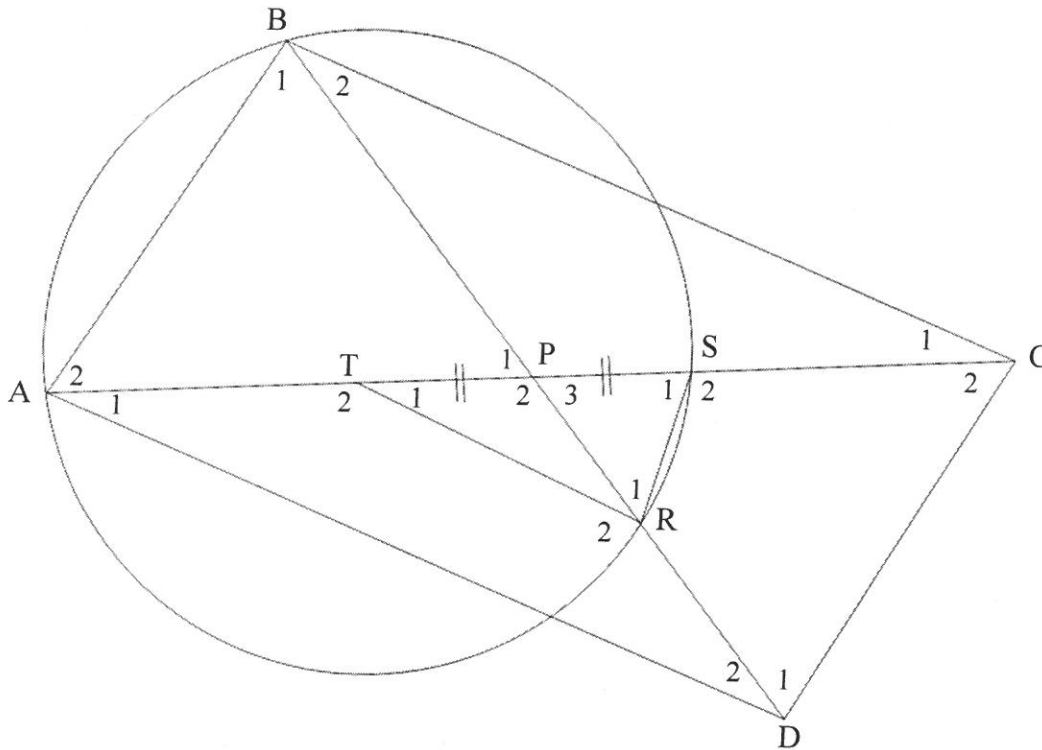
QUESTION 11

11.1 In the diagram, $\triangle ABC$ and $\triangle DEF$ are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem which states that if two triangles, $\triangle ABC$ and $\triangle DEF$, are equiangular, then $\frac{DE}{AB} = \frac{DF}{AC}$. (6)

- 11.2 In the diagram, ABCD is a parallelogram with A and B on the circle. The diagonals BD and AC intersect in P. PC and PD intersect the circle at S and R respectively. T is a point on AP such that TP = PS. TR is drawn.



- 11.2.1 Prove that:
- (a) $AT = SC$ (2)
 - (b) $\triangle PSR \parallel \triangle PBA$ (5)
- 11.2.2 If it is further given that $\frac{PR}{PA} = \frac{TR}{AD}$, prove that:
- (a) $\triangle RPT \parallel \triangle APD$ (3)
 - (b) ATRD is a cyclic quadrilateral (2)
- [18]**

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$