basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2015

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 27 pages./
Hierdie memorandum bestaan uit 27 bladsye.

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.
- Penalty of only 1 mark for incorrect rounding throughout the paper (Q1.2.1)

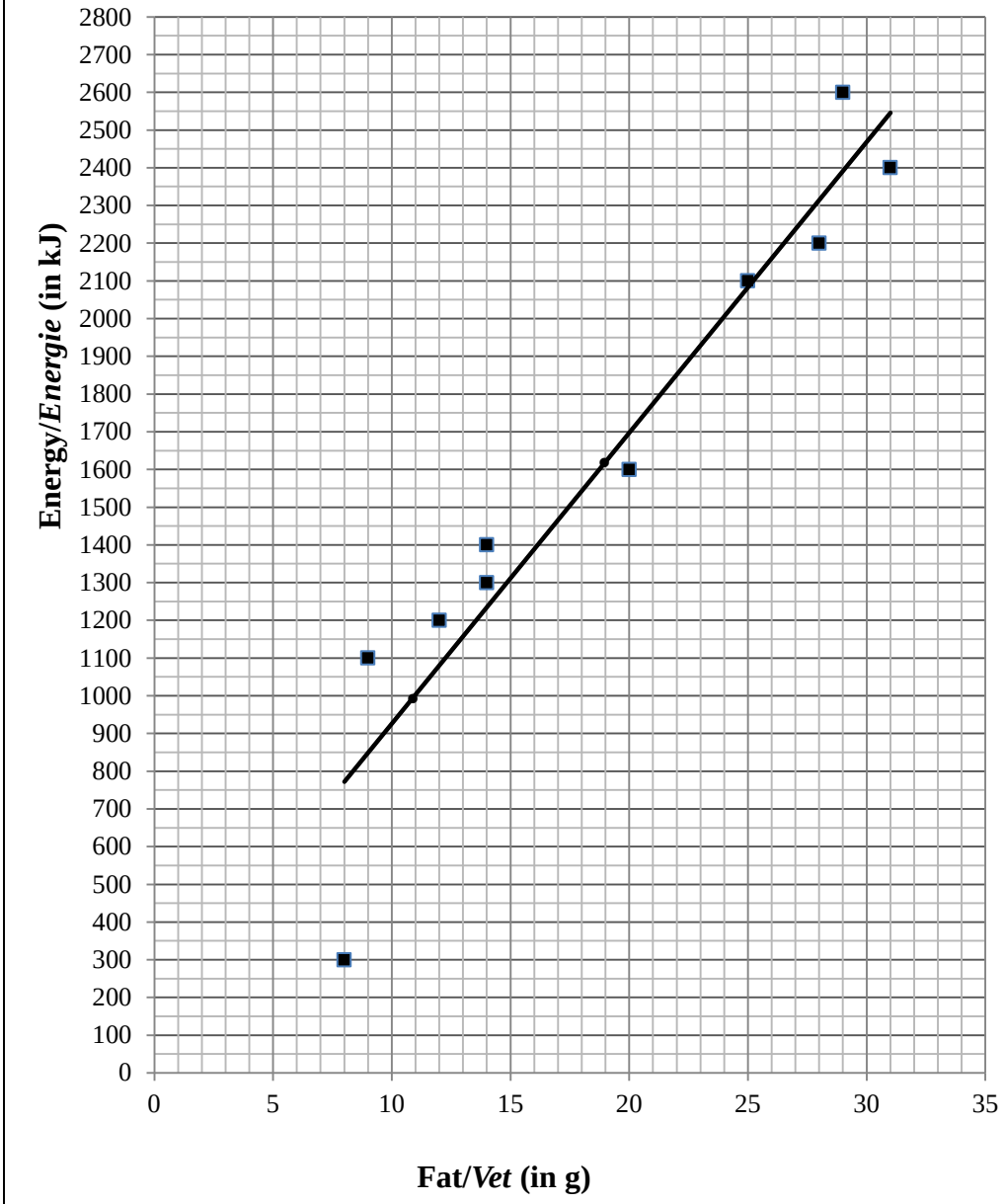
LET WEL:

- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek het en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

QUESTION/*VRAAG* 1

Fat/*Vet* (in g)	9	14	25	8	12	31	28	14	29	20
Energy/*Energie* (in kJ)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1



1.1
no marks:
0 – 2 points correctly

✓plotting
3 – 5 points correctly

✓✓plotting
6 – 9points correctly

✓✓✓plotting all 10 points correctly

geen punte:
0 – 2 punte korrek

✓ *stip 3 – 5 pte korrek*

✓✓*stip 6 –9 pte korrek*

✓✓✓ *stip al 10 pte korrek*

(3)

1.2.2

1.2.2
✓ *y* – int close to (0 ; 150)
✓ one pt close to (25 ; 2100) or (20 ; 1700)

(2)

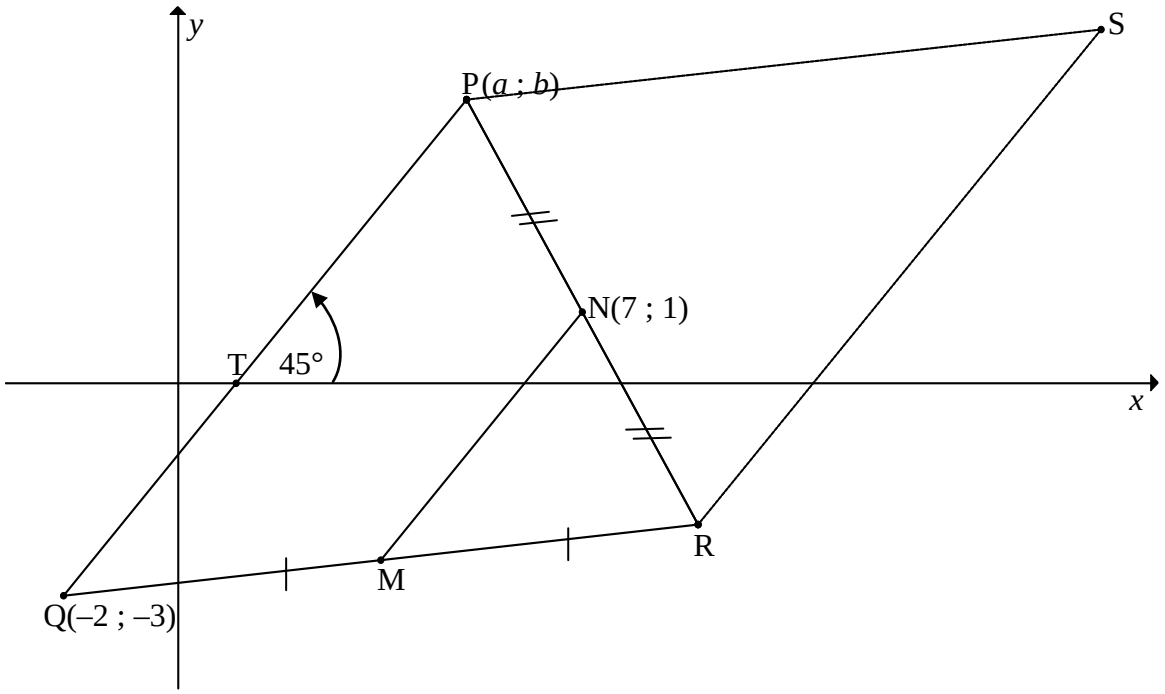
1.2.1	$\hat{y}$ = 154,60 + 77,13(18) = 1 542,94 ≈ 1 500 kJ	✓ subst ✓ answ rounded off correctly/ *antw korrek afgerond* (2)
1.3	(8 ; 300)	✓ answ/*antw* (1)
1.4	r = 0,9520... ≈ 0,95	✓✓ answ/*antw* (2)
1.5	very strong positive relationship/ *baie sterk positiewe verband*	✓ strong/ *sterk* (1) **[11]**

QUESTION/*VRAAG* 2

Sum of the values on uppermost faces/ ***Som van die waardes op boonste vlakke***	**Frequency/** ***Frekwensie***
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1	mean/*gemiddelde* $= \frac{2(0)+3(3)+4(2)+...12(1)}{30} = \frac{202}{30}$ $= 6,73$	✓202 ✓ answ/*antw* (2)
2.2	median/*mediaan* $= \frac{T_{15}+T_{16}}{2} = \frac{7+7}{2} = 7$	✓✓ answ/*antw* (2)
2.3	SD/*SA* = 2,264... ≈ 2,26	✓✓ answ/*antw* (2)
2.4	(6,73 – 2,26 ; 6,73 + 2,26) = (4,47 ; 8,99) ∴4 + 4 + 8 + 3 = 19 times/*keer*	✓lower boundary ✓upper boundary ✓ answ/*antw* (3) **[9]**

QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \tan 45°$ $= 1$	✓ $m = \tan 45°$ ✓ answ/*antw* (2)
3.2	MN || QP [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y - y_1 = m(x - x_1)$ $\therefore y - 1 = 1(x - 7)$ $\therefore y = x - 6$ **OR/*OF*** MN || PQ [midpt theorem/*midpt-stelling*] $\therefore m_{MN} = 1$ $\therefore y = mx + c$ $\therefore 1 = 1(7) + c$ $-6 = c$ $\therefore y = x - 6$	✓ S **OR** R ✓ m_{MN} ✓ subst *m* and/*en* N(7 ; 1) ✓ equation/*vgl* (4) ✓ S **OR** R ✓ m_{MN} ✓ subst *m* and/*en* N(7 ; 1) ✓ equation/*vgl* (4)
3.3	$MN = \frac{1}{2}PQ$ [midpoint theorem/midp stelling] $\therefore MN = \frac{7\sqrt{2}}{2} \approx 4{,}95$	✓ S ✓ answ/antw (2)

3.5	QN = NS [diag of ||m/hoekl van ||m] $\frac{-2+x_S}{2} = 7$ and/en $\frac{-3+y_S}{2} = 1$ $\therefore\ x_S = 16$ $\therefore\ y_S = 5$ **OR/*OF*** QN = NS [diag of ||m/hoekl van ||m] $\therefore$ by inspection/deur inspeksie: S(16 ; 5)	✓ method/metode ✓ *x*-value/waarde ✓ *y*-value/waarde (3) ✓ method/metode ✓ *x*-value/waarde ✓ *y*-value/waarde (3)
3.6	Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = b^2 + 6b + 9 + b^2 + 6b + 9$ $0 = 2b^2 + 12b - 80$ $0 = b^2 + 6b - 40$ $\therefore 0 = (b + 10)(b - 4)$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4) **OR/*OF*** Equation of/*Vgl van PQ*: $y = x + c$ $-3 = -2 + c$ $y = x - 1$ $\therefore a = b + 1$ .....(1) From distance formula/*Van afstandsformule*: $7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$ $\therefore 98 = (a+2)^2 + (b+3)^2$ .....(2) Subst (1) into (2): $98 = (b+1+2)^2 + (b+3)^2$ $98 = 2(b+3)^2$ $49 = (b+3)^2$ $\pm 7 = b + 3$ $\pm 7 - 3 = b$ $\therefore\ b = 4$ (since $b > 0$) Subst $b = 4$ into (1): $\therefore a = 4 + 1 = 5$ $\therefore$ P(5 ; 4)	✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ st form/*st vorm* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6) ✓ eq of/*vgl van* PQ ✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule* ✓ subst eq of/*vgl v.* PQ ✓ simplification/ *vereenvoudig* ✓ value of/*waarde van b* ✓ value of/*waarde van a* (6)

OR/*OF*

Equation of/*Vgl van PQ*: $y = x + c$

$-3 = -2 + c$

$y = x - 1 \qquad \therefore a = b + 1 \quad .....(1)$

From distance formula/*Van afstandsformule*:

$7\sqrt{2} = \sqrt{(a-(-2))^2 + (b-(-3))^2}$

$98 = (a+2)^2 + (a-1+3)^2$

$= 2(a+2)^2$

$\therefore a + 2 = 7$ (since/*aangesien* $a > 0$)

$\therefore a = 5$

Subst $a = 4$ into (1):

$\therefore b = 5 - 1 = 4$

$\therefore$ P(5 ; 4)

✓ eq of/*vgl van* PQ

✓ subst Q & $7\sqrt{2}$ into/*in* distance formula/ *afstandsformule*

✓ subst eq of/*vgl v.* PQ

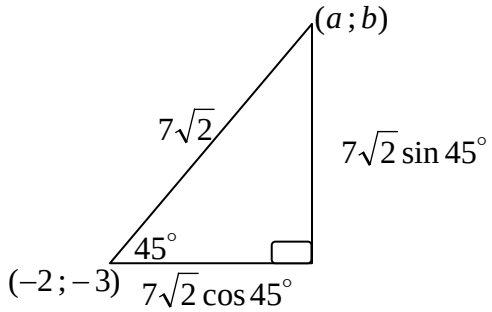
✓ simplification/ *vereenvoudig*

✓ value of/*waarde van a*

✓ value of/*waarde van b*

(6)

OR/*OF*



✓✓✓✓

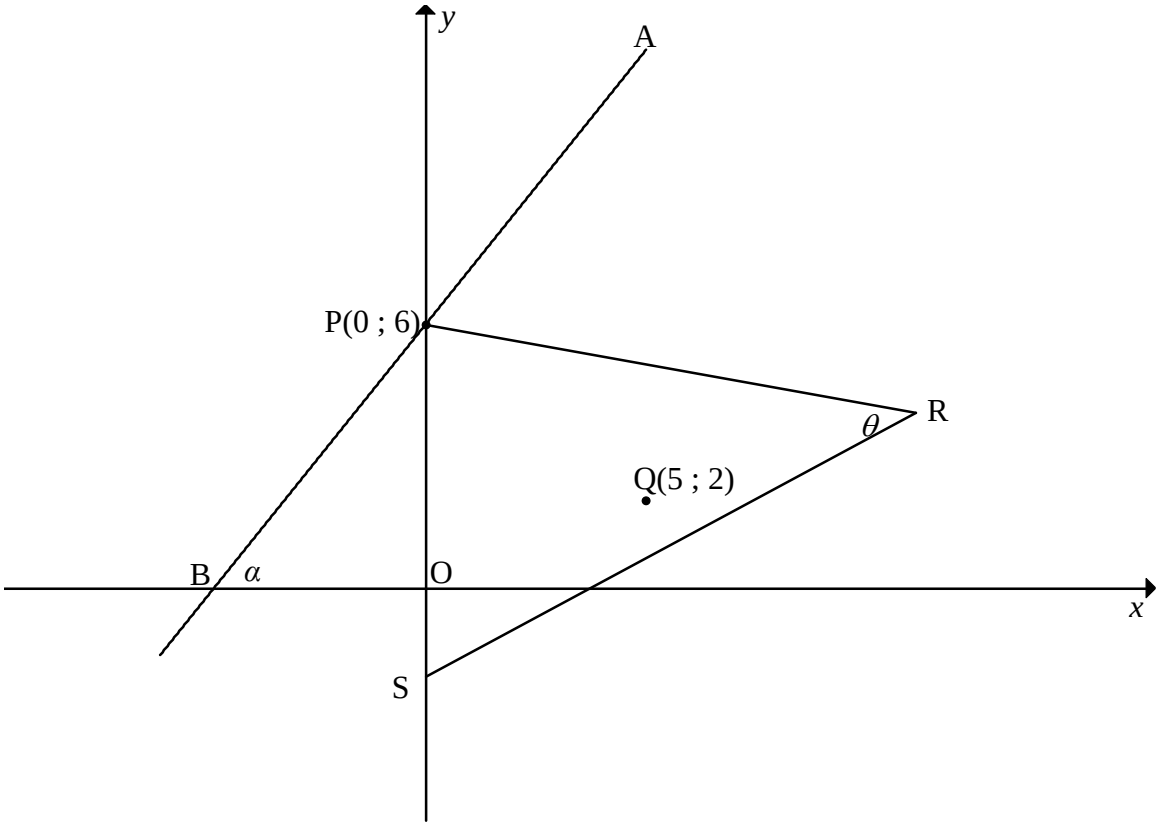
$a = -2 + 7\sqrt{2}\cos 45^\circ = 5$ ✓

$b = -3 + 7\sqrt{2}\sin 45^\circ = 4$ ✓

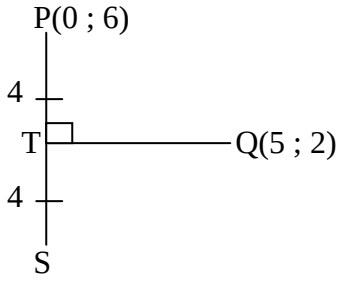
(6)

[17]

QUESTION/*VRAAG* 4

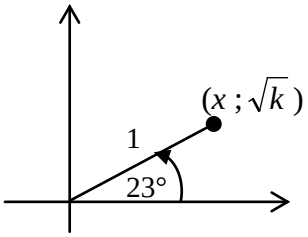


4.1	$(x-5)^2+(y-2)^2=r^2$ $(0-5)^2+(6-2)^2=r^2$ $25+16=r^2$ $41=r^2$ $\therefore\ (x-5)^2+(y-2)^2=41$ **OR/*OF*** $PQ=\sqrt{(0-5)^2+(6-2)^2}$ $=\sqrt{25+16}$ $r=\sqrt{41}$ $\therefore\ (x-5)^2+(y-2)^2=41$	✓ subst (5 ; 2) into circle eq/*in sirkelvgl* ✓ value of/*waarde van* r^2 ✓ equation/*vgl* (3) ✓ subst (5 ; 2) & (0 ; 6) into dist. form/*in afst. form* ✓ value of/*waarde van r* ✓ equation/*vgl* (3)
4.2	$(0-5)^2+(y-2)^2=41$ $25+(y-2)^2=41$ $25+y^2-4y+4=41$ $y^2-4y-12=0$ $(y-6)(y+2)=0$ $y\neq 6$ *or / of* $y=-2$ $\therefore$ S(0 ; –2) or $y=-2$	✓ $x=0$ ✓ st form/*st. vorm* ✓ answ/*antw* (neg value) (3)

	OR/*OF* $(0-5)^2+(y-2)^2 = 41$ $25+(y-2)^2 = 41$ $(y-2)^2 = 16$ $y-2 = \pm 4$ $y = 2 \pm 4$ $y \neq 6$ or/*of* $y = -2$ $\therefore$ S(0 ; –2) **OR/*OF*** Draw/*Trek* QT $\perp$ PS PT = TS [line from centre $\perp$ to chord/ *lyn van midpt* $\perp$ *koord*] PT = $y_P - y_Q = 6-2 = 4$ $y_Q - y_S = 4$ $y_S = 2-4 = -2$ $\therefore$ S(0 ; –2)  	✓ $x = 0$ ✓ square form/ *kwadraatvorm* ✓ answ/*antw* (neg value) (3) ✓ $x = 0$ ✓✓ $y = -2$ (3)
4.3	$m_{PQ} = \frac{6-2}{0-5}$ $= -\frac{4}{5}$ $m_{PQ} \times m_{APB} = -1$ [tan/*raakl* $\perp$ radius] $\therefore\ m_{APB} = \frac{5}{4}$ $\therefore y = \frac{5}{4}x + 6$	✓ subst (0 ; 6) & (5 ; 2) into grad form/*in grad. formule* ✓ m_{PQ} ✓ m_{APB} ✓ equation/*vgl* (4)
4.4	$\tan\alpha = \frac{5}{4}$ $\therefore\ \alpha = 51{,}34°$ **OR/*OF*** B(4,8 ; 0) $\therefore \tan\alpha = \frac{6}{4{,}8}$ $\therefore\ \alpha = 51{,}34°$	✓ $\tan\alpha = m_{APB}$ ✓ answ/*antw* (2) ✓ $\tan\alpha = \frac{6}{4{,}8}$ ✓ answ/*antw* (2)

4.5	$\theta = B\hat{P}S$ [tan-chord th/*raakl-koordst.*] $= 90° - \alpha$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $= 90° - 51{,}34°$ $= 38{,}66°$ **OR/*OF*** $PS = 8$ $PQ = SQ = \sqrt{41}$ $PS^2 = PQ^2 + SQ^2 - 2.PQ.SQ.\cos P\hat{Q}S$ $64 = 41 + 41 - 2.41.\cos P\hat{Q}S$ $\cos P\hat{Q}S = \frac{18}{82}$ $P\hat{Q}S = 77{,}32°$ $\theta = \frac{1}{2}P\hat{Q}S$ [$\angle$ at centre = 2 × $\angle$ circumf] $= 38{,}66°$	✓ S ✓ R ✓ $90° - \alpha$ ✓ answ/*antw* (4) ✓ correct subst into cosine rule ✓ $P\hat{Q}S = 77{,}32°$ ✓ R ✓ answ/*antw* (4)
4.6	Area $\Delta PQS = \frac{1}{2}$ PS ×height/*hoogte* $= \frac{1}{2}(8)(5)$ $= 20$ sq units/*vk eenh* **OR/*OF*** $P\hat{Q}S = 2 \times 38{,}66°$ [$\angle$at centre = 2×$\angle$ at circum/ *midpts$\angle$=2omtreks$\angle$*] $= 77{,}32°$ Area $\Delta PQS = \frac{1}{2}PQ.QS.\sin P\hat{Q}S$ $= \frac{1}{2}.\sqrt{41}.\sqrt{41}.\sin 77{,}32°$ $= 20$ sq units/*vk eenh*	✓ area formula/*e*: ΔPQS ✓ PS = 8 ✓ $\perp h = 5$ ✓ answ/*antw* (4) ✓ size of/*grootte v* $P\hat{Q}S$ ✓ area rule/*reël*: ΔPQS ✓ subst correctly/ *subst korrek* ✓ answ/*antw* (4) **[20]**

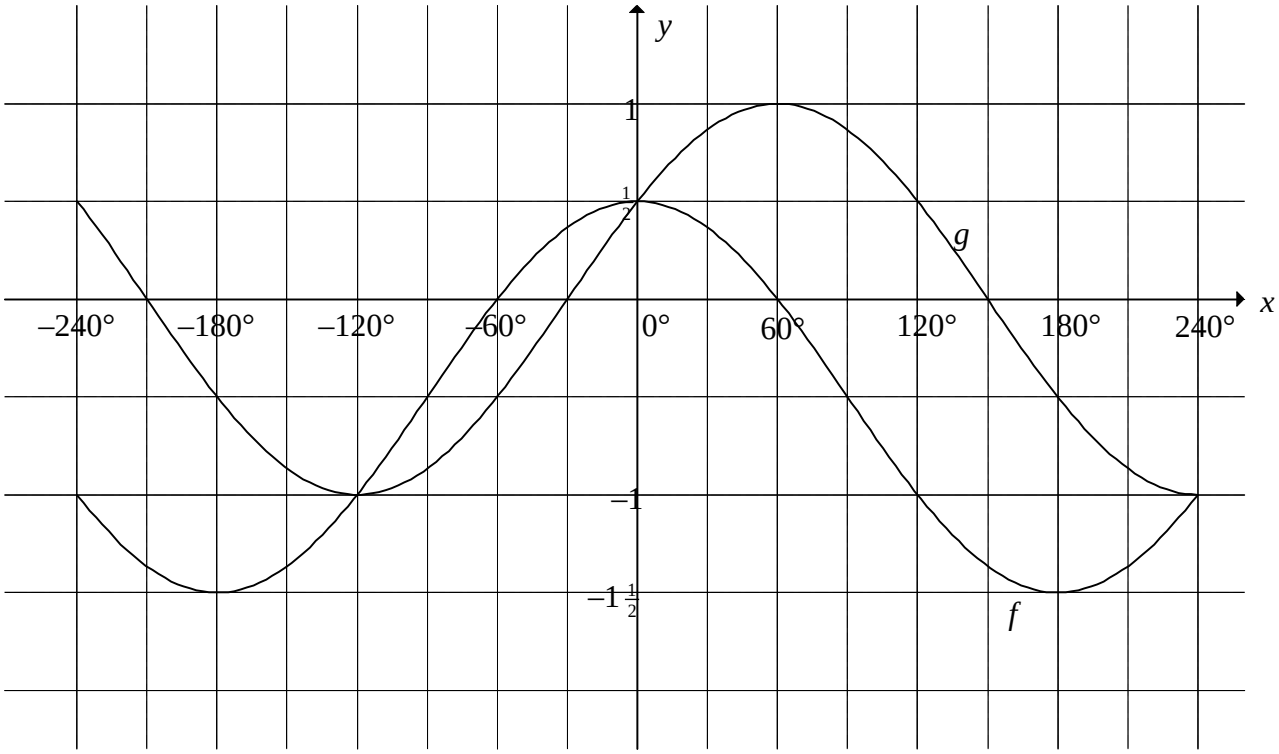
QUESTION/*VRAAG* 5

5.1.1	$\sin 203°$ $= -\sin 23°$ $= -\sqrt{k}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.1.2	$\cos^2 23° = 1 - \sin^2 23°$ $= 1 - k$ $\cos 23° = \sqrt{1-k}$ **OR/*OF*** $x^2 + (\sqrt{k})^2 = 1$ $x^2 = 1 - k$ $x = \sqrt{1-k}$ $\cos 23° = \frac{\sqrt{1-k}}{1} = \sqrt{1-k}$  	✓identity/*identiteit* ✓ $\cos^2 23°$ ito/*itv k* ✓ answ/*antw* (3) ✓ $x^2 = 1 - k$ ✓ *x* ito/*itv k* ✓ answ/*antw* (3)
5.1.3	$\tan(-23°) = -\tan 23°$ $= -\frac{\sin 23°}{\cos 23°}$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$ **OR/*OF*** $\tan(-23°) = -\tan 23°$ $= -\frac{\sqrt{k}}{\sqrt{1-k}} = -\sqrt{\frac{k}{1-k}}$	✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2) ✓ reduction/ *reduksie* ✓ answ ito/*antw itv k* (2)
5.2	$\frac{4\cos x.(-\sin x)}{\sin(30° - x + x)}$ $= \frac{-4\sin x.\cos x}{\sin 30°}$ $= \frac{-4\sin x.\cos x}{\frac{1}{2}}$ $= -8\sin x.\cos x$ $= -4(2\sin x.\cos x)$ $= -4\sin 2x$	✓ cos *x* ✓– sin *x* ✓ sin (α + β) ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

	OR/OF $\frac{4\cos x.(-\sin x)}{(\sin 30°\cos x-\cos 30°\sin x)\cos x+(\cos 30°\cos x+\sin 30°\sin x)\sin x}$ $=\frac{-4\sin x.\cos x}{(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)\cos x+(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x)\sin x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(\cos^2 x+\sin^2 x)}$ $=\frac{-2(2\sin x.\cos x)}{\frac{1}{2}(1)}$ $=-8\cos x\sin x$ $=-4(2\sin x\cos x)$ $=-4\sin 2x$	✓ $\cos x$ ✓$-\sin x$ ✓ $\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x$ ✓ $\frac{1}{2}$ ✓ double sine form / *dubbel sin form* ✓ answ/*antw* (6)

5.3	$\cos 2x - 7\cos x - 3 = 0$ $2\cos^2 x - 1 - 7\cos x - 3 = 0$ $2\cos^2 x - 7\cos x - 4 = 0$ $(2\cos x + 1)(\cos x - 4) = 0$ $\therefore \cos x = -\frac{1}{2}$ or/*of* $\cos x = 4$ (no solution) $\therefore x = 120° + n.360°$ or/*of* $x = 240° + n.360°$; $n \in Z$ **OR/OF** $\therefore x = \pm 120° + n.360°$; $n \in Z$	✓ expansion/ *uitbreiding* ✓ $2\cos^2 x - 7\cos x - 4 = 0$ ✓ factors/*faktore* ✓ $\cos x = -\frac{1}{2}$ ✓ 120° & 240° ✓ $+ n.360°$ **OR/OF** ✓ $\pm 120°$ ✓ $+ n.360°$ (6)
5.4	$\sin 3\theta = \sin(2\theta + \theta)$ $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ $= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$ $= 3(\frac{1}{3}) - 4(\frac{1}{3})^3$ $= 1 - \frac{4}{27}$ $= \frac{23}{27}$	✓ expansion of/ *uitbreiding van* $\sin(2\theta + \theta)$ ✓ expansions of $\sin 2\theta$ AND $\cos 2\theta$ ✓ $1 - \sin^2\theta$ ✓ subst ✓ answ/*antw* (5) **[24]**

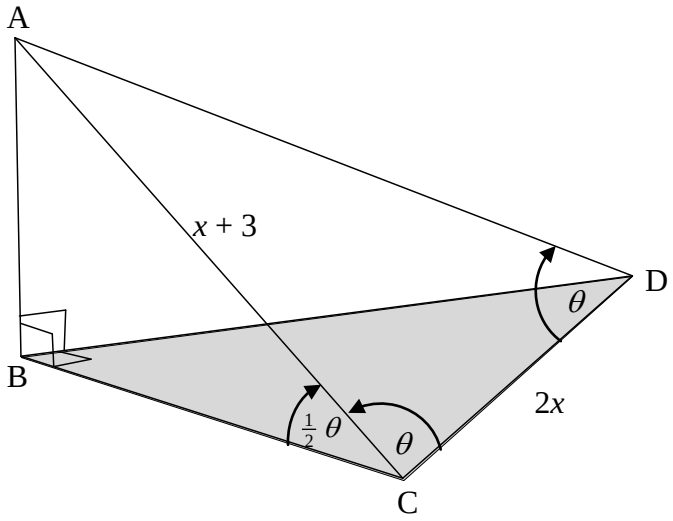
QUESTION/*VRAAG* 6

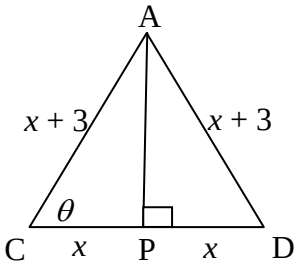


6.1	$f(x) = \cos x - \frac{1}{2}$ and/*en* $g(x) = \sin(x + 30°)$ $\therefore p = 30°$ and/*en* $q = -\frac{1}{2}$ **OR/*OF*** $\sin(60° + p) = 1$ and/*en* $\cos 0° + q = \frac{1}{2}$ $\therefore p = 30°$ $\therefore q = -\frac{1}{2}$	✓ $f(x) = \cos x - \frac{1}{2}$ ✓ $g(x) = \sin(x + 30°)$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4) ✓ $\sin(60° + p) = 1$ ✓ $\cos 0° + q = \frac{1}{2}$ ✓ value of/*waarde v p* ✓ value of/*waarde v q* (4)
6.2	$x \in (-120° ; 0°)$ **OR/*OF*** $-120° < x < 0°$	✓ critical values/ *kritiese waardes* ✓ correct interval/ *korrekte interval* (2)

6.3	The graph of *g* has to shift 60° to the left and then be reflected about the *x*-axis./*Die grafiek van g moet 60° na links skuif en dan om die x-as gereflekteer word.*	✓ 60° left/*links* ✓ reflection about *x*-axis/*refleksie om x-as* (2)
	OR/*OF* The graph of *g* must be reflected about the *x*-axis and then be shifted 60° to the left./*Die grafiek van g moet om die x-as gereflekteer word en dan met 60° na links geskuif word.*	✓ reflection about *x*-axis/*refleksie om x-as* ✓ 60° left/*links* (2)
	OR/*OF* The graph of *g* has to shift 120° to the right./*Die grafiek van g moet 120° na regs geskuif word.*	✓ ✓ 120° right/*regs* (2)
	OR/*OF* The graph of *g* has to shift 240° to the left./*Die grafiek van g moet met 240° na links geskuif word*	✓ ✓ 240° left/*links* (2) **[8]**

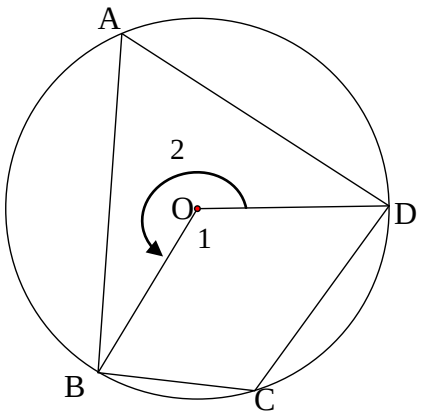
QUESTION/*VRAAG* 7



7.1	$C\hat{A}D = 180° - 2\theta$ [∠s sum of Δ/*∠e som van Δ*]	✓ answ/*antw* (1)
7.2	$\frac{\sin\theta}{x+3} = \frac{\sin(180° - 2\theta)}{2x}$ $\frac{\sin\theta}{x+3} = \frac{\sin 2\theta}{2x}$ $\frac{\sin\theta}{x+3} = \frac{2\sin\theta.\cos\theta}{2x}$ $\cos\theta = \frac{2x\sin\theta}{2(x+3)\sin\theta}$ $\cos\theta = \frac{x}{x+3}$	✓ correct subst into sine rule/*korrekte subst in sin-reël* ✓ $\sin 2\theta$ ✓ $2\sin\theta.\cos\theta$ ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* AD = $x + 3$ [sides opp = ∠s/*sye to = ∠e*] $AC^2 = AD^2 + CD^2 - 2AD.CD.\cos\theta$ $(x+3)^2 = (x+3)^2 + (2x)^2 - 2(2x)(x+3).\cos\theta$ $0 = 4x^2 - 4x(x+3)\cos\theta$ $\cos\theta = \frac{4x^2}{4x(x+3)}$ $= \frac{x}{x+3}$	✓ AD = $x + 3$ ✓ correct subst into cosine rule/*korrekte subst in cos-reël* ✓ simplification/ *vereenvoudiging* ✓ $\cos\theta$ as subject/ *as onderwerp* (4)
	OR/*OF* Draw/*Trek* AP ⊥ CD $\cos\theta = \frac{x}{x+3}$ 	✓ ✓ constr/*konstr* ✓ ✓ sketch shown/ *toon skets* (4)

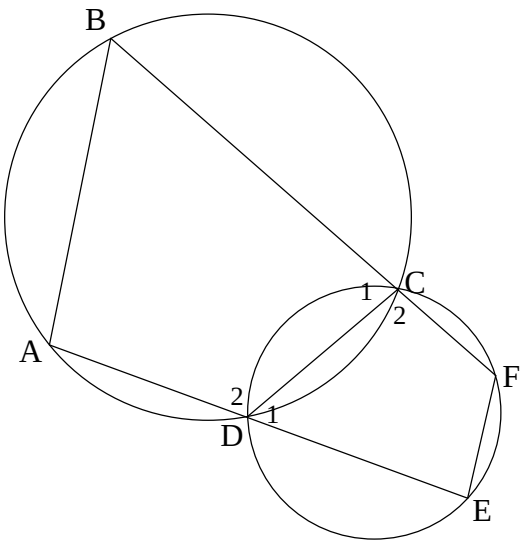
7.3	$\cos\theta = \frac{2}{5}$ $\therefore\ \theta = 66{,}42°$ In ΔABC: $\sin\frac{1}{2}\theta = \frac{AB}{AC}$ $\sin 33{,}21° = \frac{AB}{5}$ $\therefore\ AB = 5\sin 33{,}21°$ $= 2{,}74$ **OR/*OF*** $\sin\frac{\theta}{2} = \frac{AB}{5}$ $\therefore AB = 5\sin\frac{\theta}{2}$ but/*maar*: $\cos\theta = \frac{2}{5}$ $1 - 2\sin^2\frac{\theta}{2} = \frac{2}{5}$ $\sin^2\frac{\theta}{2} = \frac{3}{10}$ $\sin\frac{\theta}{2} = \sqrt{\frac{3}{10}}$ $\therefore AB = 5\sqrt{\frac{3}{10}} = \sqrt{\frac{15}{2}} = 2{,}74$	✓ $\cos\theta = \frac{2}{5}$ ✓ size of/*grootte v* θ ✓ correct ratio/ *korrekte verh* ✓ subst correctly/ *korrek* ✓ answ/*antw* (5) ✓ $AB = 5\sin\frac{\theta}{2}$ ✓ equation/*vgl* ✓ simplification/ *vereenvoudiging* ✓ value of/*waarde v* $\sin\frac{\theta}{2}$ ✓ answ/*antw* (5) **[10]**

QUESTION/*VRAAG* 8



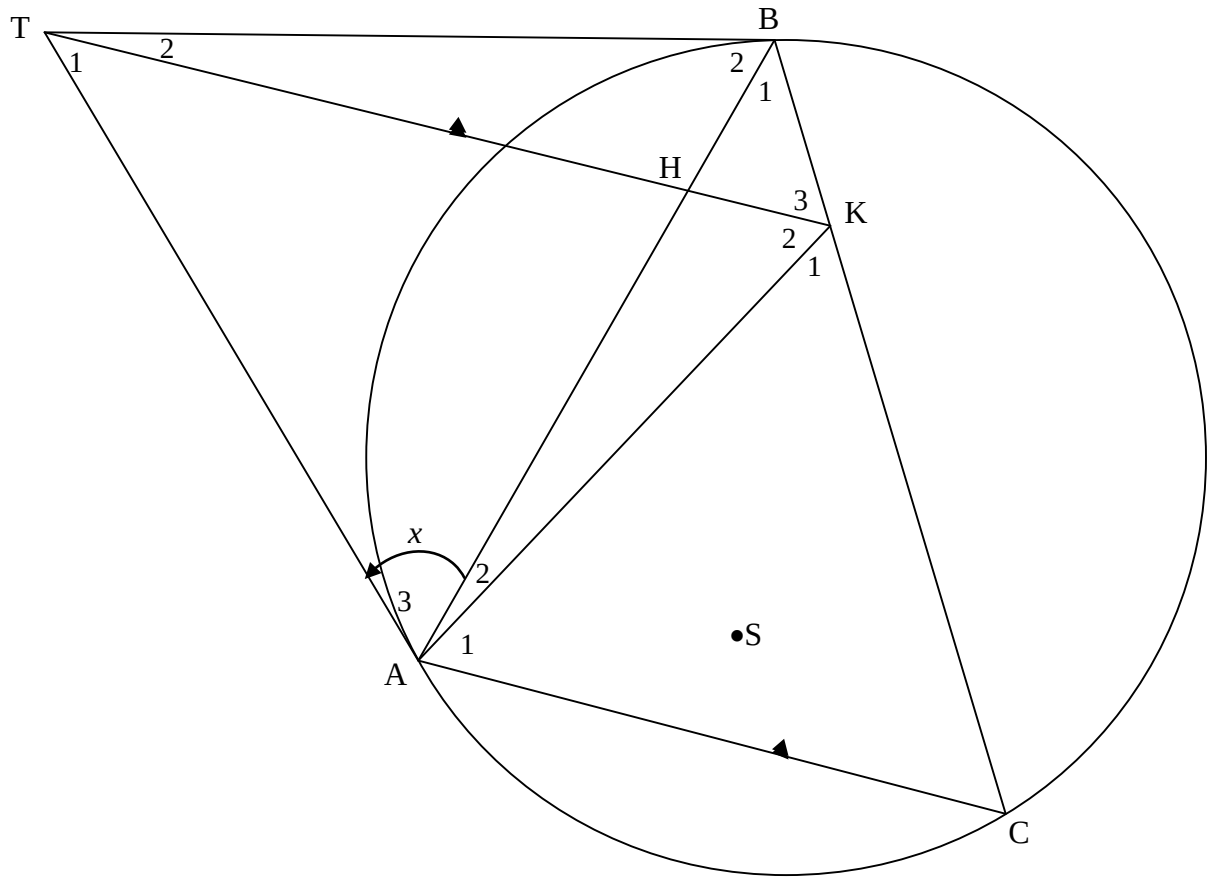
8.1.1	twice or double /*twee keer of dubbel*	✓ R (1)
8.1.2	$\hat{O}_1 = 2\hat{A}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 2\hat{C}$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s in a rev/*∠e in omw of om 'n pt*] $2\hat{A} + 2\hat{C} = 360°$ $\therefore \hat{A} + \hat{C} = 180°$ **OR/OF** Let/*Gestel* $\hat{O}_1 = 2x$ $\hat{A} = x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\hat{O}_2 = 360° - 2x$ [∠s in a rev/*∠e in omw of om 'n pt*] $\hat{C} = 180° - x$ [∠ at centre = 2×∠ at circ/*midpts∠ =2×omtreks∠*] $\therefore \hat{A} + \hat{C} = 180°$	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ S (3)

8.2



8.2	$\hat{A} = \hat{C}_2$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{E} = 180° - \hat{C}_2$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{E} = 180° - \hat{A}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5)
	OR/*OF*		
	$\hat{B} = \hat{D}_1$	[ext ∠ of cyclic quad/*buite∠ v kdvh*]	✓ S ✓ R
	$\hat{F} = 180° - \hat{D}_1$	[opp ∠s of cyclic quad/*tos∠e v kdvh*]	✓ S ✓ R
	$\therefore \hat{F} = 180° - \hat{B}$		
	$\therefore$ EF || AB	[co-interior ∠s 180°/*ko-binne∠e 180°*]	✓ R (5) **[9]**

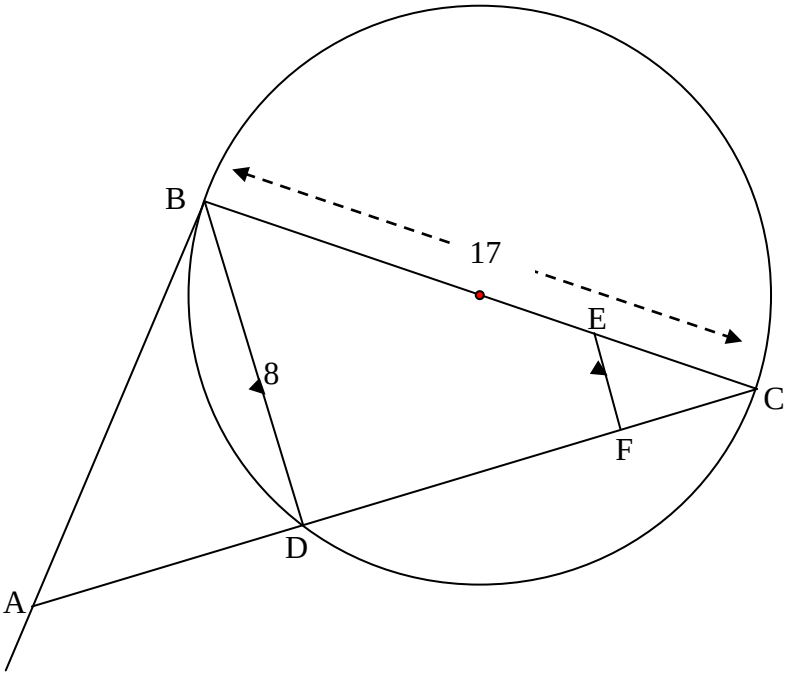
QUESTION/*VRAAG* 9



9.1	$\hat{K}_3 = \hat{C}$ $= \hat{A}_3$ $= x$	[corresp ∠s/*ooreenk ∠e* ; CA| |KT] [tan-chord th/*raakl-koordst*]	✓ S ✓ R ✓ S ✓ R (4)
9.2	$\hat{K}_3 = x = \hat{A}_3$ ∴ AKBT is cyc quad	[proved/*bewys* in 9.1] [line (BT) subtends equal ∠s/ *lyn (BT) onderspan gelyke ∠e*] **OR/OF** [converse ∠s in same segment/ *omgek ∠e in dies segment*]	✓ S ✓ R (2)
9.3	$\hat{K}_3 = \hat{C}$ $= \hat{B}_2$ $= \hat{K}_2$ ∴ TK bisects/*halveer* $A\hat{K}B$ **OR/*OF*** $\hat{K}_2 = \hat{B}_2$ $= \hat{A}_3$	[proven in 9.1] [tan-chord th/*raakl-koordst*] [∠s in the same segm/*∠e in dies segm*] [∠s in the same seg/*∠e in dies segm*] [tans from same pt; ∠s opp equal sides/ *rkle v dies pt; ∠e to gelyke sye*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R

	$\therefore \quad = \hat{K}_3$ [proven in 9.1] $\therefore$ TK bisects/*halveer* $A\hat{K}B$	(4)
9.4	$\hat{A}_3 = \hat{K}_2 = x$ [proven/*bewys*] $\therefore$ TA tangent [converse tan chord theorem OR ∠ between line and chord/ *omgekeerde raakl-kdst* ***OF*** *∠tussen lyn en koord*]	✓ S ✓ R (2)
9.5	$B\hat{S}A = B\hat{K}A = 2x$ [A,S,K & B concyclic/*konsiklies*] $A\hat{T}B = 180° - 2x$ [A,T,B & K concyclic/*konsiklies*] $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is ook konsiklies* [opp ∠s of quad = 180°/*tos ∠e van vierhoek=180°*] **OR/*OF*** A, S K and B are concyclic. A, K, B and T are concyclic. $\therefore$ A, S, B and T are concyclic. **OR/*OF*** The circle passing through points A, K and B contains the point S on the circumference (A, ,S, K and B concyclic)./*Die sirkel deur punt A, K en B bevat die punt S op die omtrek (A, S, K en B konsiklies).* The circle passing through A, K and B contains the point T on the circumference (proven in 9.2)./*Die sirkel deur punt A, K en B bevat die punt T op die omtrek (bewys in 9.2).* $\therefore$ points A, S, B and T are also concyclic/*punte A, S, B en T is konsiklies*	✓ S (both/*beide* statements/ *bewerings*) ✓ R (2) ✓ S ✓ S (2) ✓ S ✓ S (2) **[14]**

QUESTION/*VRAAG* 10



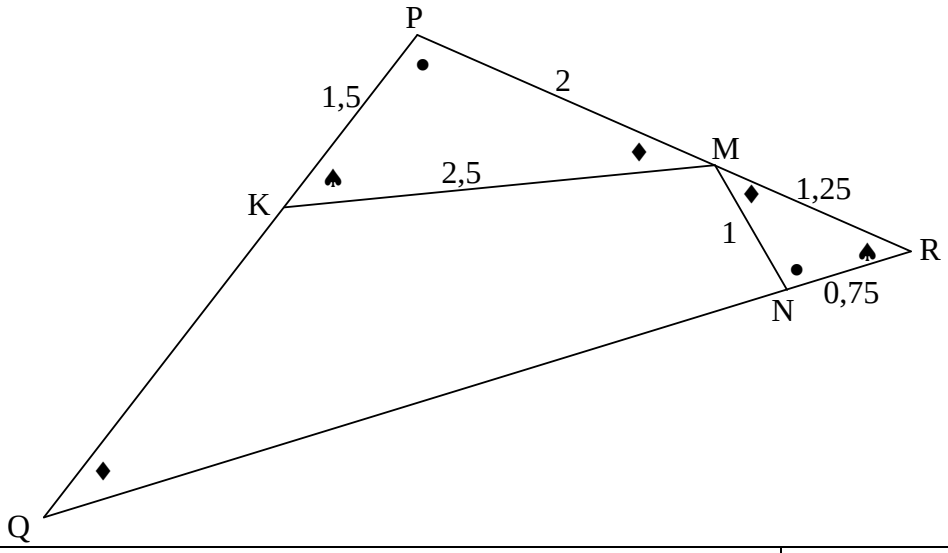
10.1	$B\hat{D}C = 90°$ $DC^2 = 17^2 - 8^2$ $= 225$ $\therefore DC = 15$	[∠ in semi circle/∠ *in halfsirkel*] [Th of/*stelling v* Pythagoras]	✓ S ✓ using/*gebruik* Pyth *korrek/* correctly ✓ answ/*antw* (3)
10.2.1	$\frac{CF}{CD} = \frac{CE}{CB}$ $\therefore \frac{CF}{15} = \frac{1}{4}$ $\therefore CF = 3,75$	[line | | one side of Δ/*lyn* | | *een sy van* Δ] **OR/OF** ΔCEF | | | ΔCBD	✓ S/R ✓ subst correctly/ *korrek* ✓ answ/*antw* (3)
10.2.2	$B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$ $\therefore$ ΔBAC | | | Δ FEC **OR/*OF*** $B\hat{D}C = 90°$ $E\hat{F}C = B\hat{D}C$ $A\hat{B}C = 90°$ In ΔBAC and/*en* Δ FEC: $A\hat{B}C = E\hat{F}C$ $\hat{C} = \hat{C}$	[∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*] [∠∠∠] [∠ in semi circle/∠ *in halfsirkel*] [corresp ∠s/*ooreenk ∠e*; EF| |BD] [tan ⊥ diameter/*raakl ⊥ middellyn*] [proven/bewys] [common/*gemeen*]	✓ S/R ✓ S ✓ R ✓ S ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S

	$B\hat{A}C = F\hat{E}C$ [$\angle$ sum in Δ/$\angle$ *som van* Δ] $\therefore \Delta BAC \mid\mid\mid \Delta FEC$	✓ S (5)
10.2.3	$EC = \frac{1}{4} \times 17 = 4,25$ $\frac{AC}{EC} = \frac{BC}{FC}$ [$\Delta BAC \mid\mid\mid \Delta FEC$] $\frac{AC}{4,25} = \frac{17}{3,75}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\cos \hat{C} = \frac{CF}{CE} = \frac{BC}{AC}$ $\therefore \frac{3,75}{4,25} = \frac{17}{AC}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\Delta BCA \mid\mid\mid \Delta DBC$ $CB^2 = CD \cdot AC$ $AC = \frac{BC^2}{DC}$ $= \frac{17^2}{15}$ $= 19,27$ or/*of* $19\frac{4}{15}$ **OR/*OF*** $\hat{C} = A\hat{B}D$ [tan-chord theorem/*rkl-kdstelling*] $\frac{AD}{8} = \tan A\hat{B}D$ $= \tan \hat{C}$ $= \frac{8}{15}$ $\therefore AD = \frac{64}{15}$ $\therefore AC = 19,27$ or/*of* $19\frac{4}{15}$	✓ length of/*lengte v* EC ✓ S ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓✓ correct ratios/ *korrekte verh's* ✓ subst correctly/ *korrek* ✓ answ/*antw* (4) ✓ S **OR** Pyth th ✓ correct ratio ✓ subst ✓ answ/*antw* (4) ✓ S ✓ correct ratio ✓ subst ✓ answ/*antw* (4)

10.2.4	AC is diameter of the circle passing through A, B and C [chord subtends 90° **OR** converse $\angle$ in semi circle] *AC is middellyn van die sirkel wat deur die punte A, B en C gaan* [*koord onderspan 90° **OF** omgek $\angle$ in halfsirkel*] $\therefore$ radius = $\frac{1}{2}\times 19{,}27 = 9{,}63$ or/*of* $9\frac{19}{30}$ or/*of* $\frac{1}{2}$AC	✓ S/R ✓ answ/*antw* (2) **[17]**

QUESTION/*VRAAG* 11

11.1	equiangular or similar/*gelykhoekig of gelykvormig*	✓ answ/*antw* (1)



11.2.1	$\frac{KP}{RN}=\frac{1,5}{0,75}=2$; $\frac{PM}{NM}=\frac{2}{1}=2$; $\frac{KM}{RM}=\frac{2,5}{1.25}=2$ $\therefore \frac{KP}{RN}=\frac{PM}{NM}=\frac{KM}{RM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** $\frac{RN}{KP}=\frac{0,75}{1,5}=\frac{1}{2}$; $\frac{NM}{PM}=\frac{1}{2}$; $\frac{RM}{KM}=\frac{1,25}{2,5}=\frac{1}{2}$ $\therefore \frac{RN}{KP}=\frac{NM}{PM}=\frac{RM}{KM}$ $\therefore \Delta KPM \mid\mid\mid \Delta RNM$ [Sides of Δ in prop/*sye v Δ eweredig*] **OR/*OF*** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] $\cos P\hat{K}M = \frac{1,5}{2,5}=\frac{3}{5}$ and $\cos \hat{R} = \frac{0,75}{1,25}=\frac{3}{5}$ $\therefore P\hat{K}M = \hat{R}$ In ΔKPM and ΔRNM $P\hat{K}M = \hat{R}$ [proved] $\hat{P} = M\hat{N}R$ [proved] $\therefore \Delta KPM \mid\mid \Delta RNM$ [∠;∠;∠ OR 3rd ∠]	✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓✓✓ all 3 statements/ *al 3 bewerings* (3) ✓ $\hat{P} = M\hat{N}R$ ✓ $P\hat{K}M = \hat{R}$ ✓[∠;∠;∠ OR 3rd ∠] (3)

11.2.2	$P\hat{K}M = \hat{R}$ [$\Delta KPM \mid\mid\mid \Delta RNM$] $\therefore$ $\hat{P}$ is common/*gemeen* $\therefore$ $\Delta RPQ \mid\mid\mid \Delta KPM$ [$\angle\angle\angle$] $\frac{RP}{KP} = \frac{RQ}{KM}$ [$\Delta RPQ \mid\mid\mid \Delta KPM$] $\therefore \frac{3,25}{1,5} = \frac{RQ}{2,5}$ $\therefore RQ = \frac{2,5 \times 3,25}{1,5} = 5,42$ or $5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$ **OR/OF** $R\hat{N}M = \hat{P}$ [$\Delta KPM \mid\mid\mid \Delta RNM$] $\therefore$ $\hat{R}$ is common/*gemeen* $\therefore$ $\Delta RNM \mid\mid\mid \Delta RPQ$ [$\angle\angle\angle$] $\frac{RP}{RN} = \frac{RQ}{RM}$ [$\Delta RNM \mid\mid\mid \Delta RPQ$] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42$ or $5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$ **OR/*OF*** In ΔMNR: $1,25^2 = 1^2 + 0,75^2 = 1,5625$ $\therefore M\hat{N}R = 90°$ [converse Pyth theorem] In ΔPKM: $2,5^2 = 1,5^2 + 2^2 = 6,25$ $\therefore \hat{P} = 90°$ [converse Pyth theorem] In ΔMNR and ΔQPR $\angle R$ is common $M\hat{N}R = \hat{P} = 90°$ $\therefore \Delta MNR \mid\mid\mid \Delta QPR$ [$\angle\angle\angle$] $\frac{RP}{RN} = \frac{RQ}{RM}$ [$\Delta RNM \mid\mid\mid \Delta RPQ$] $\therefore \frac{3,25}{0,75} = \frac{RQ}{1,25}$ $\therefore RQ = 5,42$ or $5\frac{5}{12}$ $\therefore NQ = 5,42 - 0,75 = 4,67$ or $4\frac{2}{3}$	✓ S ✓ $\Delta RPQ \mid\mid\mid \Delta KPM$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) ✓ S ✓ $\Delta RNM \mid\mid\mid \Delta RPQ$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) ✓ S ✓ $\Delta MNR \mid\mid\mid \Delta QPR$ ✓ S ✓ subst correctly/ *korrek* ✓ $RQ = 5\frac{5}{12}$ ✓ NQ = answ/*antw* (6) **[10]**

TOTAL/*TOTAAL*: 149