basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 26 pages.
Hierdie memorandum bestaan uit 26 bladsye.

NOTE:

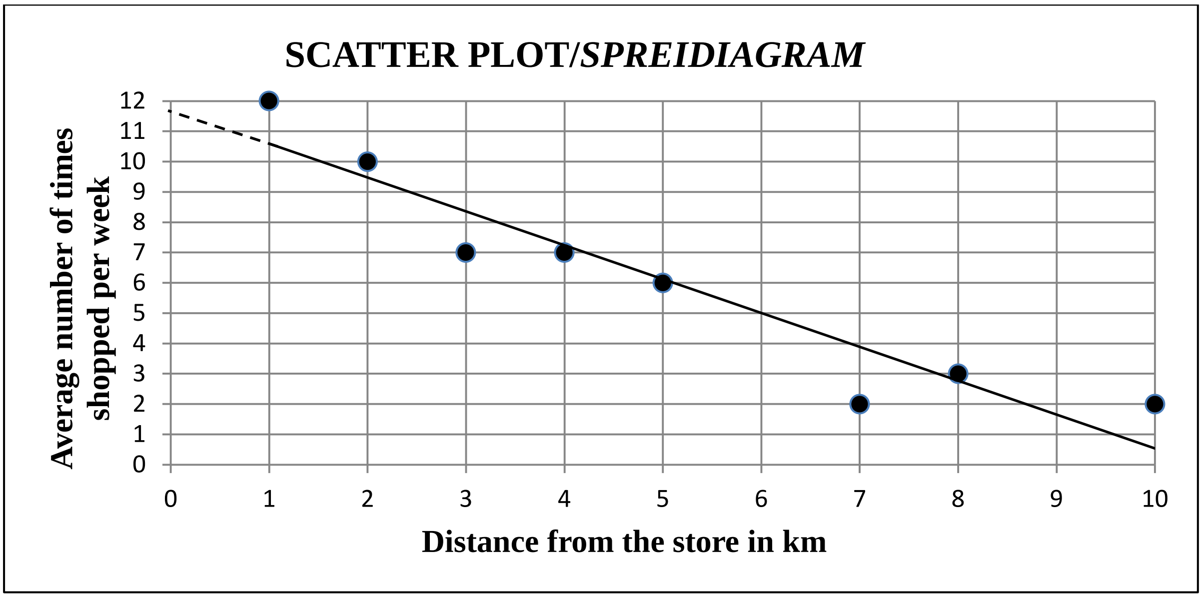
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

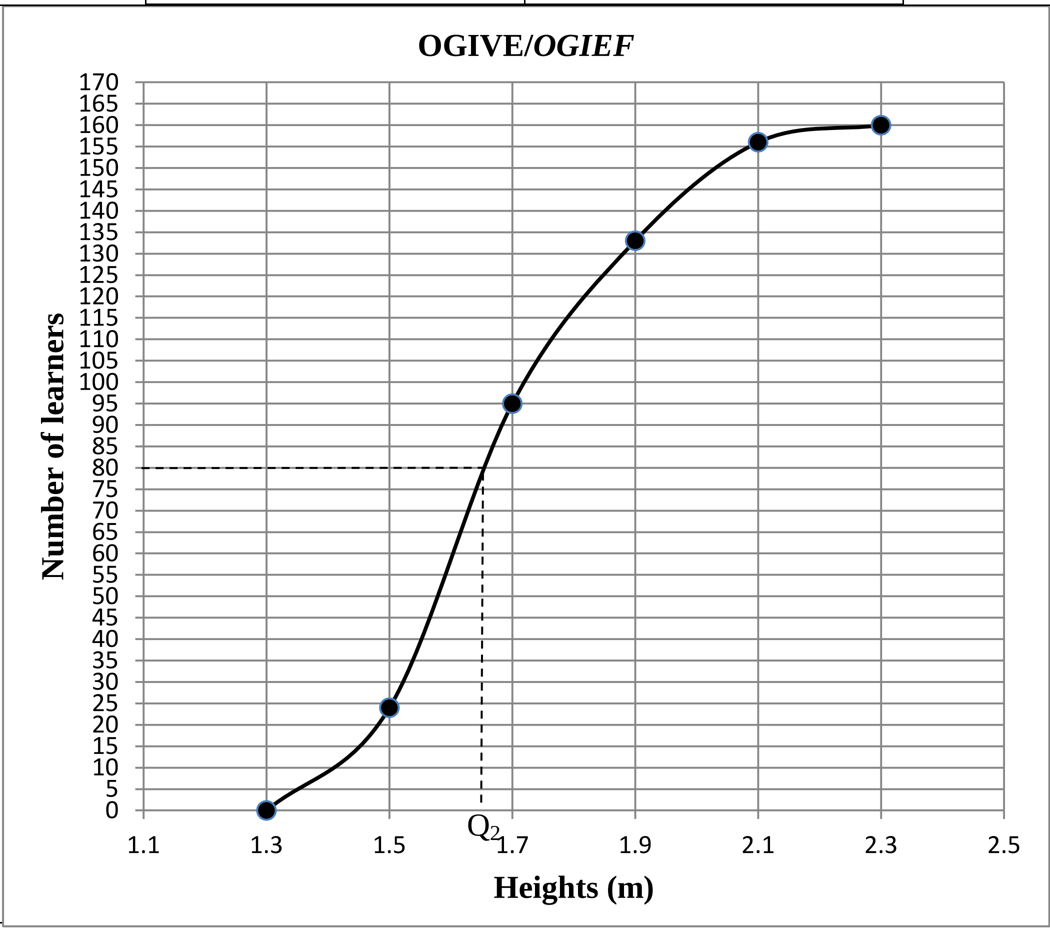
QUESTION/*VRAAG* 1

Distance from the store in km *Afstand vanaf die winkel in km*	1	2	3	4	5	7	8	10
Average number of times shopped per week *Gemiddelde aantal keer wat kopers die winkel per week besoek*	12	10	7	7	6	2	3	2

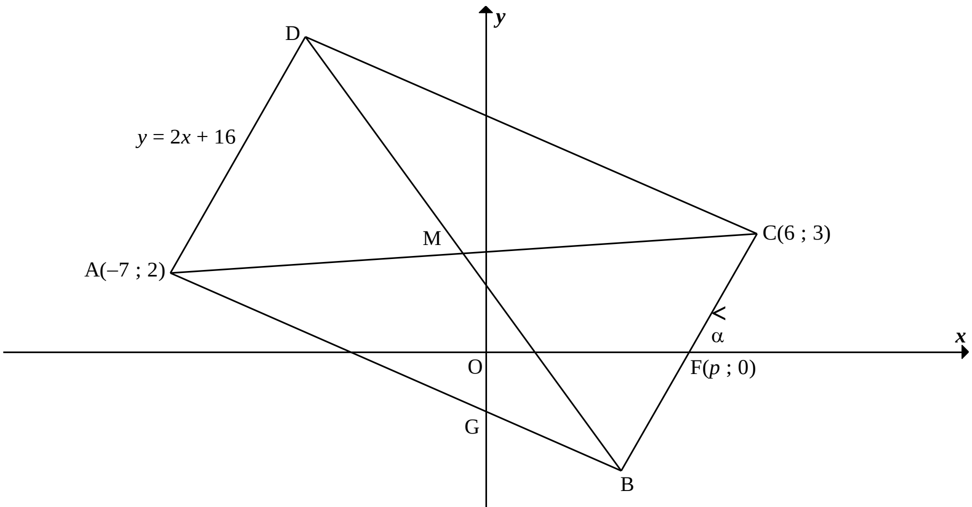


1.1	Strong/*Sterk*	✓ (1)
1.2	–0,95 (–0,9462..)	✓ (1)
1.3	$a = 11{,}71$ (11,7132…) $b = -1{,}12$ (–1,1176…) $\hat{y} = -1{,}12x + 11{,}71$	✓ value of *a* ✓ value of *b* ✓ equation/*vgl* (3)
1.4	$\hat{y} = -1{,}12(6) + 11{,}71$ = 5 times	✓ substitition ✓ answer (2)
1.5	On scatter plot/*Op spreidiagram*	✓✓ A line close to any 2 of the following points: (5 ; 6) or (10 ; $\frac{1}{2}$) or (6 ; 5) or (0 ; 11,7) (2) **[9]**

QUESTION/*VRAAG* 2

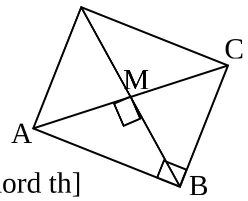
<table>
<tr><td>2.1</td><td>Positively skewed OR skewed to the right/positief skeef OF skeef na regs</td><td>✓ answer
(1)</td></tr>
<tr><td>2.2</td><td>Range/Omvang = 2,21 – 1,39 = 0,82 m</td><td>✓ subtract values
✓answer
(2)</td></tr>
<tr><td>2.3</td><td>
<table>
<tr><th>Intervals
Klasse</th><th>Cumulative frequency
Kumulatiewe frekwensie</th></tr>
<tr><td>$1,3 \leq x < 1,5$</td><td>24</td></tr>
<tr><td>$1,5 \leq x < 1,7$</td><td>95</td></tr>
<tr><td>$1,7 \leq x < 1,9$</td><td>133</td></tr>
<tr><td>$1,9 \leq x < 2,1$</td><td>156</td></tr>
<tr><td>$2,1 \leq x < 2,3$</td><td>160</td></tr>
</table>
</td><td>✓95 , 133, 156
✓160
(2)</td></tr>
<tr><td>2.4</td><td>
</td><td>✓ upper limits / boonste limiete
✓ cum f/ kum f
✓ shape/ vorm
✓ grounded geanker
(4)</td></tr>
<tr><td>2.5</td><td>method (using 80 to determine the height)
1,65 (accept any value between 1,6 and 1,69)</td><td>✓ method
✓ answer
(2)</td></tr>
<tr><td>2.6.1</td><td>The mean would change by 0,1 m
Die gemiddelde sal met 0,1 m verander</td><td>✓ answer
(1)</td></tr>
<tr><td>2.6.2</td><td>No influence/change as there is no difference in variation of data./Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.</td><td>✓ answer
(1)
[13]</td></tr>
</table>

QUESTION/*VRAAG* 3

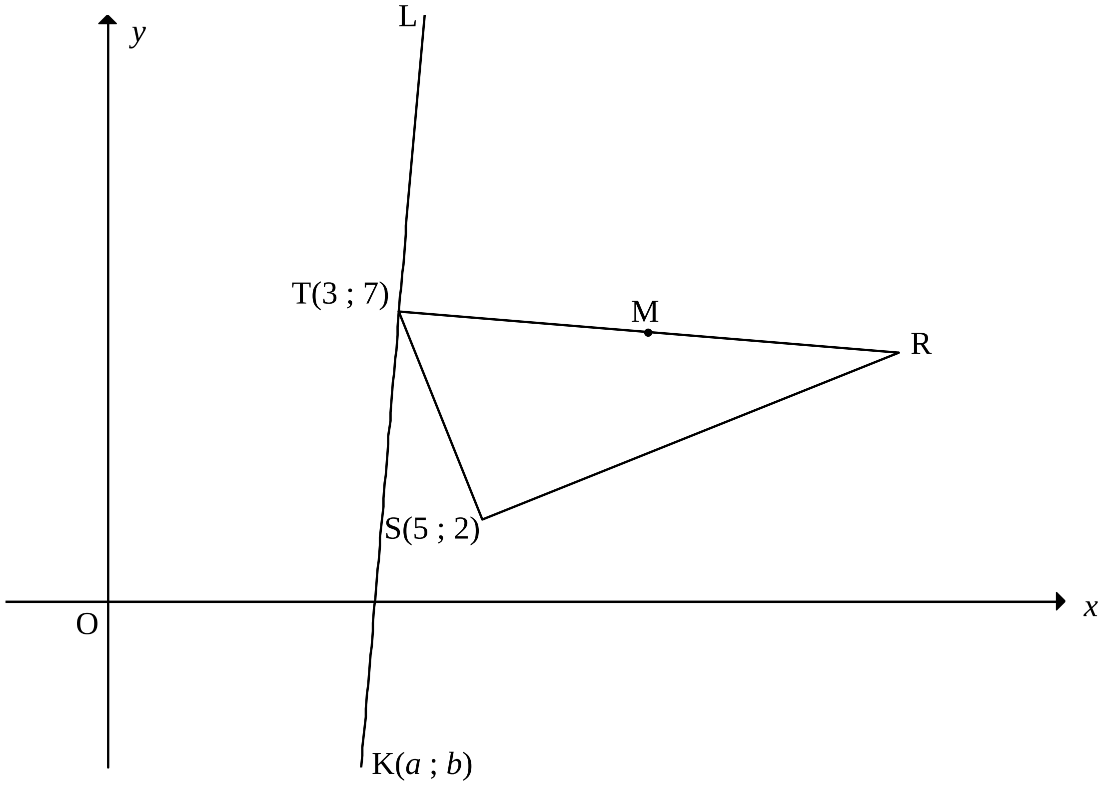


3.1	M = Midpt of AC [diags of rectangle bisect/ *hoekl v reghoek halveer*] $= M\left(\frac{-7+6}{2};\frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2};\frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ **OR/*OF*** $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓answer (1) ✓answer (1)
3.3	$m_{AD} = m_{BC}$ [AD | | BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ **OR/OF** $y - y_1 = 2(x - x_1)$ $C(6;3)$ $y - 3 = 2(x-6)$ $\therefore y = 2x - 9$ $but\ y = 0$ $\therefore x = 4\frac{1}{2} = p$ **OR/OF**	✓ $m_{BC} = 2$ ✓ equating ✓answer (3) ✓ $m_{BC} = 2$ ✓ substituting (6 ; 3) ✓answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2}$ $\therefore p = \frac{9}{2}$	✓ $m_{BC} = 2$ ✓ substituting ✓answer (3)
3.4	DB =AC [diag of rectangle = / *hoekl v reghoek* =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6+7)^2 + (3-2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore$ DB = $\sqrt{170}$ or 13,04	✓ substitution ✓ length of AC ✓ AC = BD (3)
3.5	$\tan\alpha = m_{BC} = 2$ $\therefore \alpha = 63{,}43°$	✓ $\tan\alpha = m_{BC}$ ✓ $\alpha = 63{,}43°$ (2)
3.6	In quadrilateral OFBG: $O\hat{F}B = 63{,}43°$ [vert opp ∠s/*regoorst* ∠*e*] $F\hat{O}G = G\hat{B}F = 90°$ $\therefore O\hat{G}B = 360° - [90° + 90° + 63{,}43°]$ [sum ∠s quad/*som* ∠*e vierh* = 360°] $\therefore O\hat{G}B = 116{,}57°$ **OR/*OF*** $m_{AB} = -\frac{1}{2}$ $90° + O\hat{G}A = 153{,}43°$ $\therefore O\hat{G}A = 63{,}43°$ $O\hat{G}B = 180° - 63{,}43°$ $= 116{,}57°$ **OR/*OF*** $F\hat{O}G = G\hat{B}F = 90°$ $\therefore$GOFB is cyc quad $O\hat{G}B = 180° - 63{,}43°$ [∠s of cyc quad = 180°] $= 116{,}57°$ **OR/OF** $O\hat{F}B = 63{,}43°$ $X\hat{O}G = F\hat{B}G = 90°$ $\therefore$ OGBF is a cyclic quad $\therefore O\hat{G}B = 180° - 63{,}43°$ $O\hat{G}B = 116{,}57°$	✓ size of $O\hat{F}B$ ✓ S ✓ answer (3) ✓ $m_{AB} = -\frac{1}{2}$ ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3) ✓ S ✓ S ✓ answer (3)

3.7	$M\left(-\frac{1}{2};\frac{5}{2}\right)$ is the centre/*is die middelpunt* $r=\frac{\sqrt{170}}{2}$ = radius [BD is diameter/*middellyn*] $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\left(\frac{\sqrt{170}}{2}\right)^2=\frac{85}{2}=42,5$	✓ M is centre ✓ $r=\frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$C\hat{B}M=B\hat{A}M=45°$ [diag of square bisect ∠s/*hoekl v vierk halv ∠e*] ∴ BC will be a tangent [converse tan chord th/*omgekeerde raakl-koordst*] **OR/*OF***  $A\hat{M}B=90°$ [diag of square bisect ⊥] ∴ AB is diameter BC ⊥ AB ∴ BC is tangent [line ⊥ radius *or* converse tan-chord th]	✓S ✓ R (2) ✓S ✓ R (2) **[19]**

QUESTION/*VRAAG* 4



4.1	$\angle$ in semi circle/ $\angle$ at centre = 2$\angle$ on circle $\angle$ *in halfsirkel* /$\angle$ by middelpt = 2$\angle$ op sirkel	✓ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	✓ substitution ✓ m_{TS} (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$ **OR/*OF***	✓ m_{RS} ✓substitution m and (5 ; 2) ✓ equation (3)

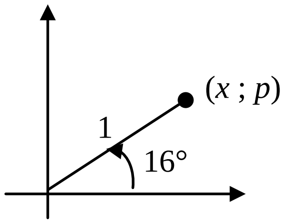
	$m_{TS} \times m_{RS} = -1$ [TS ⊥ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	✓ m_{RS} ✓substitution *m* and (5 ; 2) ✓ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ **OR/OF** $TM = \sqrt{(3-9)^2 + \left(7 - 6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	✓ *r* ✓ answer (2) ✓ substitution ✓ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark) **OR/*OF*** $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ **OR/OF**	✓ M ✓ *x* coordinate ✓ *y* coordinate (3) ✓ M ✓ *x* coordinate ✓ *y* coordinate (3)

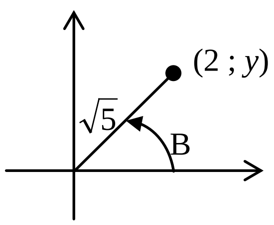
	$m_{TM} = \dfrac{9-3}{6\frac{1}{2}-7} = -\dfrac{1}{12}$ $TM: \quad 7 = -\dfrac{1}{12}(3)+c \quad y = -\dfrac{1}{12}x + \dfrac{29}{4}$ ....(1) $SR: \quad y = \dfrac{2}{5}x$ ....(2) $\dfrac{2}{5}x = -\dfrac{1}{12}x + \dfrac{29}{4}$ $\dfrac{29}{60}x = \dfrac{29}{4}$ $\therefore x = 15$ $\therefore y = \dfrac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$\text{ST} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\text{ST} = \sqrt{(5-3)^2+(2-7)^2}$ $\text{ST} = \sqrt{4+25} = \sqrt{29}$ $\sin \text{R} = \dfrac{\text{TS}}{\text{TR}} = \dfrac{\sqrt{29}}{\sqrt{145}}$ *or* $\dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$ *or* 0,45 **OR/OF** $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ area of $\Delta\text{TSR} = \dfrac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$ $29 = \dfrac{1}{2}(\sqrt{145})(2\sqrt{29})\sin R$ $\sin R = \dfrac{\sqrt{5}}{5}$ *or* $\dfrac{1}{\sqrt{5}}$	✓substitution ✓ answer ✓ ratio (3) ✓area ✓ rule ✓ ratio (3)
4.4.4	$m_{\text{TR}} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ **OR/*OF*** $m_{\text{TR}} = \dfrac{7-6}{3-15} = -\dfrac{1}{12}$ $m_{\text{TR}} \times m_{\text{KTL}} = -1$ [$r \perp$ tangent] $m_{\text{KTL}} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x-3)$ $y = 12x - 29$ substitute $\text{K}(a;b)$: $b = 12a - 29$ **OR/*OF***	✓ $m_{\text{TR}} = -\dfrac{1}{12}$ ✓ $m_{\text{KTL}} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \dfrac{7-6\frac{1}{2}}{3-9} = -\dfrac{1}{12}$ $m_{TR} \times m_{KTL} = -1$ [$r \perp$ tangent] $\dfrac{b-7}{a-3} = 12$ $b-7 = 12(a-3)$ $b = 12a - 29$	✓ $m_{TR} = -\dfrac{1}{12}$ ✓ $m_{KTL} = 12$ ✓substitution $(3\,;7)$ & $(a\,;b)$ (3)
	OR/*OF* $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = (15-3)^2 + (6-7)^2 + (a-3)^2 + (b-7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	✓subst into Pyth ✓multiplication ✓simplification (3)
4.4.5	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a - 29 - 7)^2 = 145$ $(a-3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \dfrac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2$ or $a = 4$ $\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore K(2\,;-5)$ **OR/*OF***	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓standard form ✓ subst into formula or factorise ✓ values of a ✓ value of b (6)

	TK = TR $\sqrt{(a-3)^2+(b-7)^2}=\sqrt{145}$ $(a-3)^2+(b-7)^2=145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2+(12a-29-7)^2=145$ $(a-3)^2+(12a-36)^2=145$ $(a-3)^2+144(a-3)^2=145$ $(a-3)^2=1$ $a-3=\pm1$ $a=2$ or 4 $\therefore b = 12(2) - 29 = -5$ or $b = 12(4) - 29 = 19$ $\therefore$ K(2 ; –5)	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ $(a-3)^2=1$ ✓ ±1 ✓ values of a ✓ value of b (6)
	OR/*OF* $KR^2=TR^2+TK^2$ $(a-15)^2+(b-6)^2=145+145$ $(a-15)^2+(12a-29-6)^2=290$ $(a-15)^2+(12a-35)^2=290$ $a^2-30a+225+144a^2-840a+1225=290$ $145a^2-870a+1160=0$ $a^2-6a+8=0$ $\therefore(a-2)(a-4)=0$ $a=2$ or $a=4$ $\therefore b=12(2)-29$ or $b=12(4)-29$ $=-5$ $=19$ K(2 ; –5)	✓ substitution ✓ substitution of $b = 12a - 29$ ✓standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

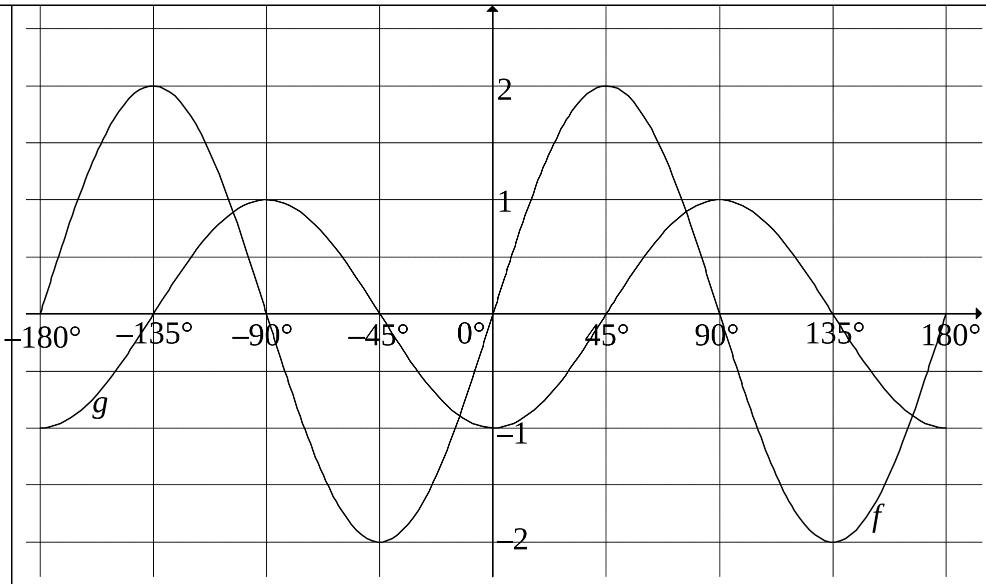
QUESTION/*VRAAG* 5

5.1.1	$\sin 196° = -\sin 16°$ $= -p$	✓reduction ✓answer (2)
5.1.2	$\cos 16° = \sqrt{1-\sin^2 16°}$ $= \sqrt{1-p^2}$ **OR/*OF***  $x^2 + p^2 = 1$ $x = \sqrt{1-p^2}$ $\therefore \cos 16° = \dfrac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$	✓statement ✓answer (2) ✓x in terms of p ✓answer (2)
5.2	$\sin(A+B) = \cos[90° - (A+B)]$ $= \cos[(90° - A) - B]$ $= \cos(90° - A)\cos B + \sin(90° - A)\sin B$ $= \sin A\cos B + \cos A\sin B$	✓co-ratio ✓correct form ✓expansion (3)
5.3	$\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A).\cos(90°+A)}$ $= \dfrac{\sqrt{\sin^2 2A}}{\cos A.(-\sin A)}$ $= \dfrac{\sin 2A}{\cos A.(-\sin A)}$ $= \dfrac{2\sin A\cos A}{\cos A.(-\sin A)}$ $= -2$ **OR/OF** $\dfrac{\sqrt{1-\cos^2 2A}}{\cos(-A)\cos(90°+A)} = \dfrac{\sqrt{1-(2\cos^2 A-1)^2}}{\cos A.-\sin A}$ $= \dfrac{\sqrt{1-(4\cos^4 A - 4\cos^2 A + 1)}}{\cos A.-\sin A} = \dfrac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A.-\sin A}$ $= \dfrac{\sqrt{4\cos^2 A(1-\cos^2 A)}}{\cos A.-\sin A} = \dfrac{\sqrt{4\cos^2 A\sin^2 A}}{\cos A.-\sin A}$ $= \dfrac{2\cos A.\sin A}{\cos A.-\sin A}$ $= -2$ **OR/OF**	 ✓$\sqrt{\sin^2 2A}$ ✓$\cos A$ ✓$-\sin A$ ✓$2\sin A\cos A$ ✓ answer (5) ✓$2\cos^2 A - 1$ ✓$\cos A$ ✓$-\sin A$ ✓identity ✓ answer (5)

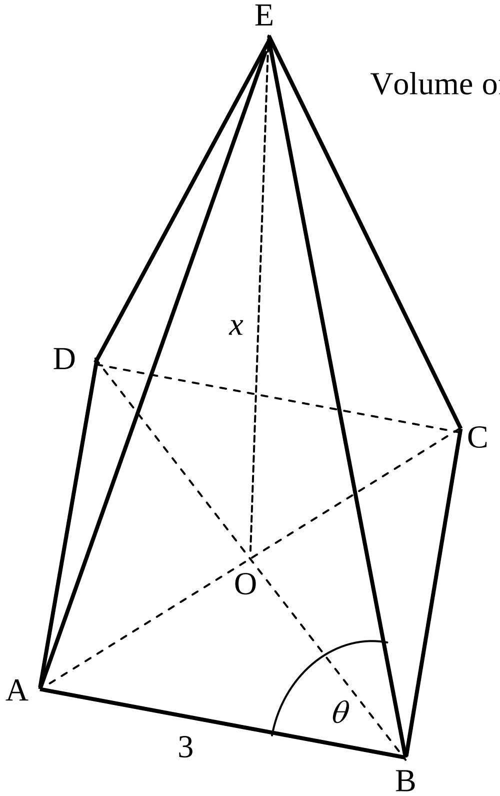
	$\dfrac{\sqrt{1-(1-2\sin^2 A)^2}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{1-(1-4\sin^2 A+4\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{\sqrt{4\sin^2 A(1-\sin^2 A)}}{\cos A.-\sin A}$ $=\dfrac{2\sin A\sqrt{\cos^2 A}}{\cos A.-\sin A}$ $=-2$	✓$1-2\sin^2 A$ ✓ cosA ✓ −sinA ✓identity ✓ answer (5)
5.4.1	$\cos 2B=\frac{3}{5}$ $2\cos^2 B-1=\frac{3}{5}$ $\cos^2 B=\frac{4}{5}$ $\therefore\ \cos B=\sqrt{\frac{4}{5}}$ *or* $\frac{2}{\sqrt{5}}$ *or* $\frac{2\sqrt{5}}{5}$ $[\,0°\le B\le 90°]$ **OR/*OF*** $\cos B=\dfrac{\sqrt{\cos 2B+1}}{2}$ $=\dfrac{\sqrt{\frac{3}{5}+1}}{2}$ $=\dfrac{2\sqrt{5}}{5}$	✓ identity ✓ value of $\cos^2 B$ ✓ answer (3) ✓ $=\dfrac{\sqrt{\cos 2B+1}}{2}$ ✓ value of $\cos^2 B$ ✓ answer (3)
5.4.2	$\sin^2 B=1-\cos^2 B$ $=1-\left(\frac{2}{\sqrt{5}}\right)^2$ $=\frac{1}{5}$ $\therefore\ \sin B=\frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$ **OR/*OF*** $(2)^2+y^2=(\sqrt{5})^2$ $4+y^2=5$ $y^2=1$ $y=1$ $\therefore\ \sin B=\frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$  	✓ $\sin^2 B=\frac{1}{5}$ ✓ answer (2) ✓ $y=1$ ✓ answer (2)

	OR/*OF* $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}}$ *or* $\frac{\sqrt{5}}{5}$	✓ $\sin^2 B = \frac{1}{5}$ ✓ answer (2)
5.4.3	$\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}}$ *or* $\frac{\sqrt{10}}{10}$ **OR/*OF*** $\cos(B + 45°) = \cos B.\cos 45° - \sin B.\sin 45°$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}}$ *or* $\frac{\sqrt{10}}{10}$	✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4) ✓ expansion ✓ $\left(\frac{1}{\sqrt{2}}\right)$ ✓ $\left(\frac{2}{\sqrt{5}}\right)$ & $\left(\frac{1}{\sqrt{5}}\right)$ ✓answer (4)
		[21]

QUESTION/*VRAAG* 6

6.1	 	✓ *x*- intercepts/ *afsnitte* ✓ *y*- intercept/ *afsnit* ✓ turning pts/ *draaipte* (3)
6.2	$f(x)-3=2\sin 2x-3$ ∴ maximum value = 2 – 3 = –1	✓ ✓ answer (2)
6.3	$2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.180°;k\in Z$ $x=76{,}72°+k.90°;k\in Z$ *or* $x=-13.28°+k.90°,k\in Z$ **OR/*OF*** $2\sin 2x=-\cos 2x$ $\tan 2x=-\frac{1}{2}$ $ref\angle=26{,}57°$ $2x=153{,}43°+k.360°$ *or* $333{,}43°+k.360°,k\in Z$ $x=76{,}72°+k.180°$ *or* $166{,}72°+k.180°\ ;k\in Z$	✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ or – 26,56° ✓ 76,72° or – 13,28° ✓ $k.90°;k\in Z$ (4) ✓ $\tan 2x=-\frac{1}{2}$ ✓ $2x=153{,}43°$ & 333,43° ✓ 76,72° & 166,72° ✓ $k.180°;k\in Z$ (4)
6.4	$x\in(-103{,}28°;-13{,}28°)$ **OR/*OF*** $-103{,}28°<x<-13{,}28°$	✓✓ values ✓notation (3) ✓✓ values ✓notation (3) **[12]**

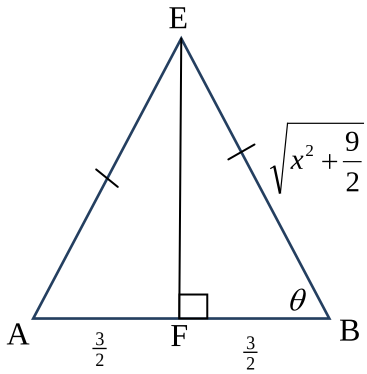
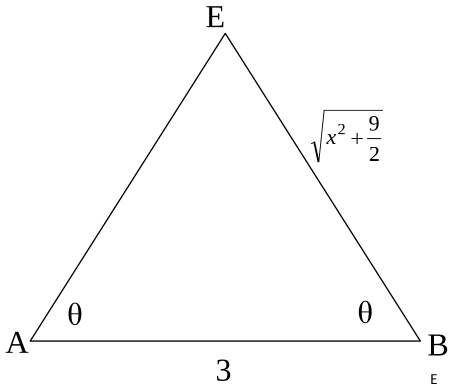
QUESTION/*VRAAG* 7



$$\text{Volume of pyramid} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$$

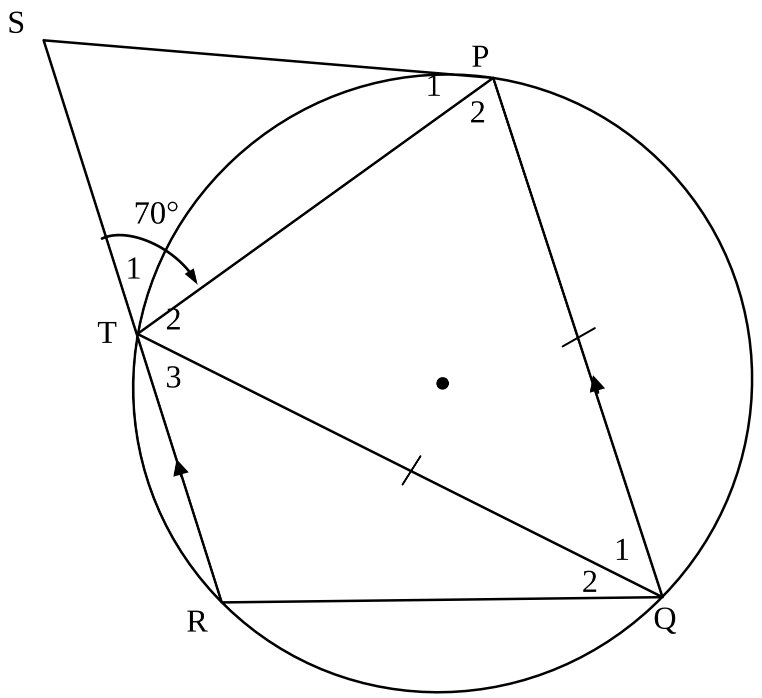
7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2}DB = \frac{\sqrt{18}}{2}$ $or \frac{3}{\sqrt{2}} or \frac{3\sqrt{2}}{2} or 2{,}12$ **OR/*OF*** $\sin 45° = \frac{OB}{3}$ $OB = 3\sin 45°$ $OB = \frac{3\sqrt{2}}{2} or \frac{3}{\sqrt{2}} or 2{,}12$ **OF/OR** $\cos 45° = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}} or \frac{3\sqrt{2}}{2} or 2{,}12$	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)

	OR/OF $\hat{AOB} = 90°$ (diagonals bisect ⊥) $OB = OA$ $AB^2 = AO^2 + BO^2$ [pyth] $\therefore AB^2 = 2OB^2$ $2OB^2 = 3^2$ $\therefore OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ OB = OA ✓ Pyth ✓ answer (3)
7.2	$BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\cos\theta = \frac{AB^2 + EB^2 - AE^2}{2AB.EB} = \frac{AB^2}{2AB.EB}$ [EB = AE] $\cos\theta = \frac{AB}{2EB}$ $\cos\theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ **OR/*OF*** $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $AE^2 = AB^2 + EB^2 - 2AB.EB\cos\theta$ $\left(\sqrt{x^2 + \frac{9}{2}}\right)^2 = 9 + \left(\sqrt{x^2 + \frac{9}{2}}\right)^2 - 2(3)\left(\sqrt{x^2 + \frac{9}{2}}\right).\cos\theta$ $\cos\theta = \frac{9}{6\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$	✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ cos θ as subject ✓ simplification (5) s ✓ substitution into Pyth ✓ length of BE ✓ correct cosine rule ✓ substituting ✓ cos θ as subject (5)

	OR/*OF* $BE^2 = EO^2 + OB^2$ (Pyth) $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos\theta = \frac{\frac{3}{2}}{\sqrt{x^2+\frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2+\frac{9}{2}}}$ 	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5)
	OR/OF $\hat{E} = 180° - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2+\frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\sin\theta\cos\theta} = \frac{\sqrt{x^2+\frac{9}{2}}}{\sin\theta}$ $\therefore \frac{3}{2\cos\theta} = \sqrt{x^2+\frac{9}{2}}$ $\cos\theta = \frac{3}{2\sqrt{x^2+\frac{9}{2}}}$ 	✓ $\hat{E} = 180° - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2\sin\theta\cos\theta$ (5)
7.3	$\text{Volume} = \frac{1}{3}(\text{area of base}) \times (\perp \text{ height})$ $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos\theta = \frac{3}{2\sqrt{25+\frac{9}{2}}}$ $\therefore \theta = 73{,}97°$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) **[12]**

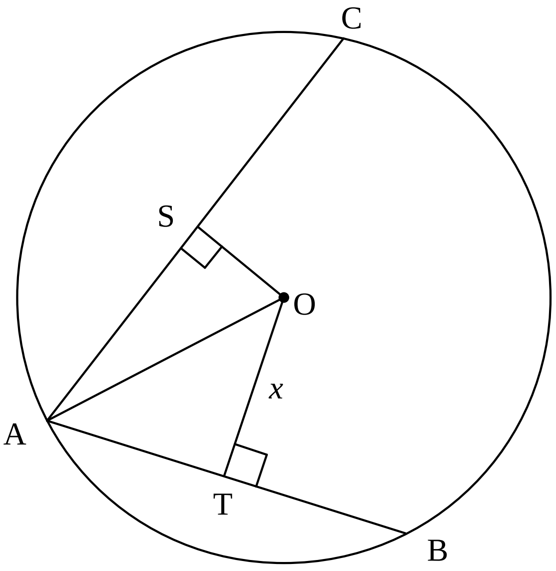
QUESTION/*VRAAG 8*

8.1



8.1.1	Alternate angles / *verwiss hoeke*, PQ ∥ SR	✓ R (1)
8.1.2(a)	$\hat{T}_2 = 70°$ [∠s opp = sides/∠*e teenoor* = *sye*] $\therefore \hat{Q}_1 = 180° - 2(70°)$ [∠s/*e* Δ = 180°] $= 40°$	✓ S ✓R ✓ answer (3)
8.1.2(b)	$\hat{P}_1 = 40°$ [tangent chord th/*raakl-koordst*]	✓ S ✓R (2)

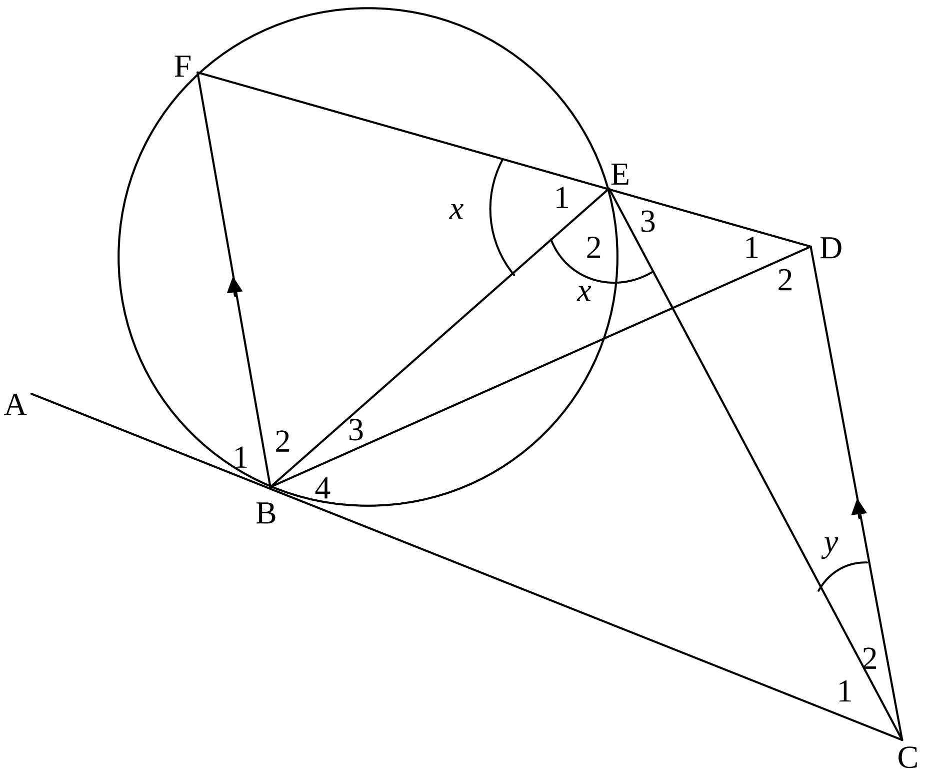
8.2



8.2.1	AT = 20 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225+400}$ $= 25$ **OR/OF** Let OS = 7, then OT = 15 In ΔAOT: $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS: $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore$ OA = 25 **OR/OF**	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓conclusion (5)

	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] Let OT = 15x. Then OS = 7x But AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore$ AO = $\sqrt{225 + 400}$ = 25	✓ equating ✓ AS = 24 ✓ substitution ✓ $x = 1$ ✓ radius (5)
	OR/OF AS = 24 [line from centre ⊥ to chord/*lyn vanaf midpt ⊥ koord*] $AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ AS = 24 ✓ substitution OS = $\frac{7}{15}$OT ✓ equating ✓ subst Pyth ✓ radius (5) **[12]**

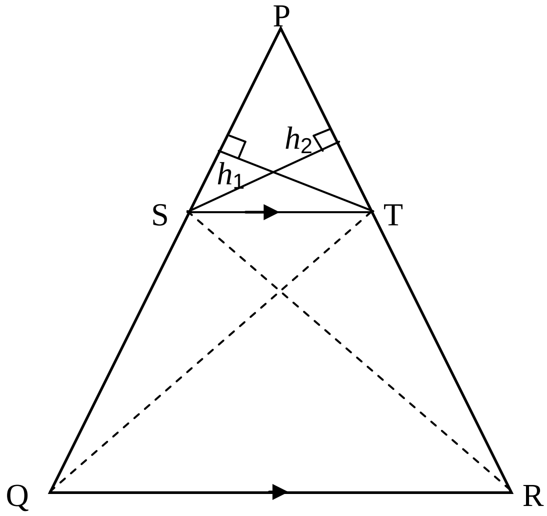
QUESTION/*VRAAG* 9



9.1.1	tangent chord theorem/*raaklyn-koordstelling*	✓ R (1)
9.1.2	corresponding/*ooreenkomstige* $\angle$s/*e*; FB || DC	✓ R (1)
9.2	$\hat{E}_1 = B\hat{C}D$ $\therefore$ BCDE = cyclic quad [converse ext $\angle$ cyc quad/*omgek: buite$\angle$kdvh*]	✓ S ✓ R (2)
9.3	$\hat{D}_2 = \hat{E}_2$ [$\angle$s in the same segment/$\angle$*e in dies segment*] $\hat{D}_2 = F\hat{B}D$ [alt $\angle$s, BF || CD/*verwiss* $\angle$*e*, *BF* || *CD*]	✓ S ✓ S (2)
9.4	$\hat{B}_3 = y$ *OR* $\hat{B}_3 = \hat{C}_2$ [$\angle$s in the same segment/$\angle$*e in dies segment*] $\hat{B}_2 = x - y$ *OR* $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ **OR/*OF*** In ΔBFE and ΔBEC $\hat{E}_1 = \hat{E}_2$ [= x] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \Delta BFE /// \Delta CBE$ [$\angle, \angle, \angle$] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S (3) ✓ identifying Δ's ✓ S ✓ S (3) **[9]**

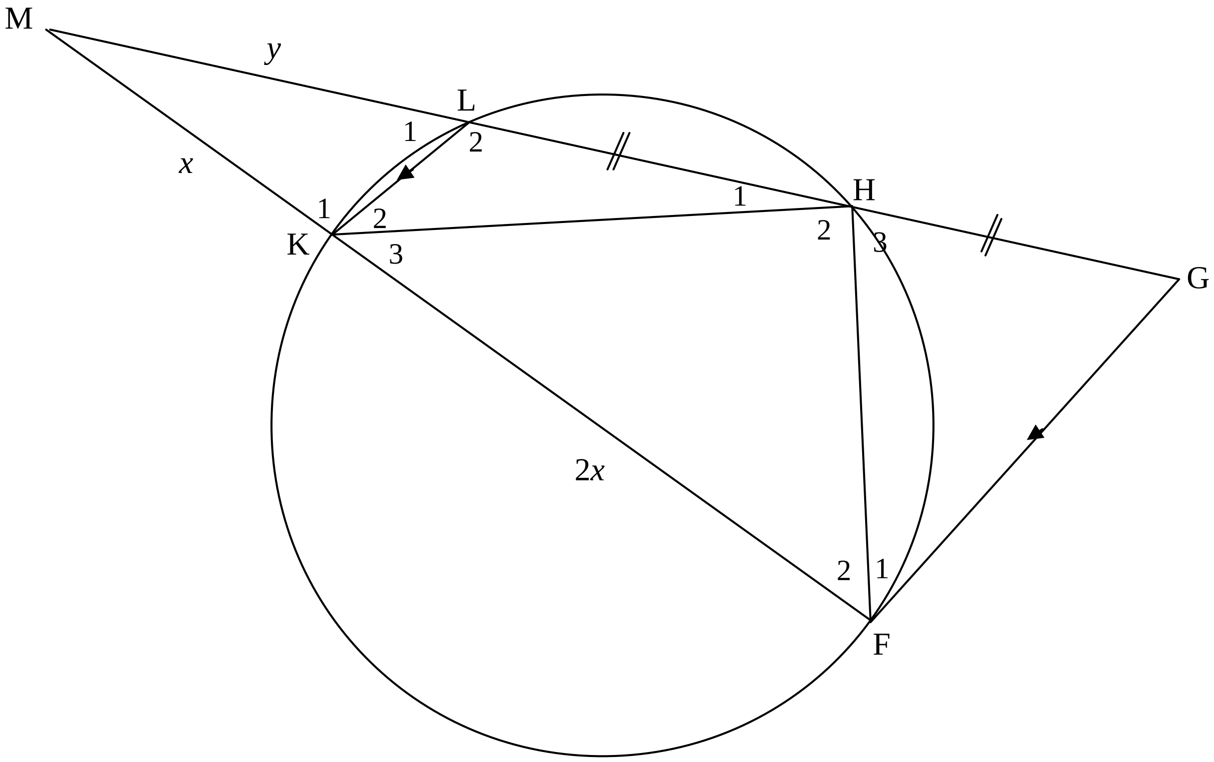
QUESTION/*VRAAG* 10

10.1



10.1		
	Constr : Join S to R and T to Q and draw h_1 from S ⊥ PT and h_2 from T ⊥ PS/ *Verbind SR en TQ en trek h_1 van S ⊥ PT en h_2 van T ⊥ PS*]	✓ constr/*konstruksie*
	Proof :	
	$\dfrac{\text{area }\Delta PST}{\text{area }\Delta QST}=\dfrac{\frac{1}{2}PS\times h_2}{\frac{1}{2}SQ\times h_2}=\dfrac{PS}{SQ}$ equal altitudes	✓ $\dfrac{\text{area }\Delta PST}{\text{area }\Delta QST}=\dfrac{\frac{1}{2}PS\times h_2}{\frac{1}{2}SQ\times h_2}$
	$\dfrac{\text{area }\Delta PST}{\text{area }\Delta STR}=\dfrac{\frac{1}{2}PT\times h_1}{\frac{1}{2}TR\times h_1}=\dfrac{PT}{TR}$ equal altitudes	✓ $\dfrac{\text{area }\Delta PST}{\text{area }\Delta STR}=\dfrac{PT}{TR}$
	area ΔPST = area ΔPST [common]	
	But area ΔQST = area ΔSTR [same base, height; ST ∥ QR]	✓ S ✓ R
	$\therefore \dfrac{\text{area }\Delta PST}{\text{area }\Delta QST}=\dfrac{\text{area }\Delta PST}{\text{area }\Delta STR}$	
	$\therefore \dfrac{PS}{SQ}=\dfrac{PT}{TR}$	✓ S
		(6)

10.2



10.2.1	Corresponding/*Ooreenkomstige* ∠s/*e*; GF || LK	✓ R (1)
10.2.2(a)	$\frac{GL}{LM} = \frac{FK}{KM}$ OR $\frac{GL}{y} = \frac{2x}{x}$ [prop theorem/*eweredighst*; GF || LK] $\frac{2GH}{y} = \frac{2x}{x}$ [LH = HG] $\therefore GH = y$	✓S ✓ R ✓ GL = 2GH (3)

10.2.2(b)	$\overleftarrow{K}_1 = \hat{GFM}$ [corresponding/*ooreenkomst* ∠ s; GF ‖ LK] $\hat{LKM}$ or $\overleftarrow{K}_1 = \hat{MHF}$ [ext ∠ cyclic quad/*buite∠koordevh*] $\hat{MHF} = \hat{GFM}$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $\hat{MHF} = \hat{GFM}$ [proven/*bewys*] ∴ ΔMFH ‖‖ ΔMGF [∠∠∠] **OR/*OR*** $\overleftarrow{K}_1 = \hat{GFM}$ [corresponding/*ooreenkomst* ∠ s; GF ‖ LK] $\hat{LKM}$ or $\overleftarrow{K}_1 = \hat{MHF}$ [ext ∠ cyclic quad/*buite∠koordevh*] $\hat{MHF} = \hat{GFM}$ In ΔMFH and ΔMGF: $\hat{M} = \hat{M}$ [common/*gemeen*] $\hat{MHF} = \hat{GFM}$ [proven/*bewys*] $\hat{F}_2 = \hat{G}$ [∠s of Δ = 180°] ∴ ΔMFH ‖‖ ΔMGF	✓S ✓ R ✓ S ✓ S ✓ R (5) ✓S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$ [‖‖ Δs] $= \frac{3x}{2y}$	✓S ✓R (2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$ [‖‖ Δs] $\frac{3x}{2y} = \frac{3y}{3x}$ [from 10.2.2(*c*)] $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	✓ S ✓substitution ✓ simplification (3) **[20]**
	TOTAL MARKS	**150**