Mathematics Form 4 Paper 1 June 2015

1 (a) 1. $(2x-1)(x+5)=0$

$x=\frac{1}{2}$ ✓ or -5 ✓ (2)

2. $(2x+3)(x-1)=2x$

$2x^2+x-3-2x=0$ ✓m

$2x^2-x-3=0$

$(2x-3)(x+1)=0$ ✓ca.

$x=\frac{3}{2}$ ✓ or -1. ✓ ca. (3)

3. $\sqrt{2x+5}=x-5$

$2x+5=x^2-10x+25$ ✓m

$x^2-12x+20=0$. ✓a

$(x-10)(x-2)=0$ ✓a

$x=10$ or 2. ✓ca

check: $x=10$ $\sqrt{25}=10-5=5$

$x=2$ $\sqrt{9}\neq 2-5$.

$x=10$ only solution ✓m (5)

4. $2x^2-4x+1$

$x=\frac{4\pm\sqrt{4^2-8}}{4}$ ✓, m

$=\frac{4\pm\sqrt{8}}{4}=\frac{2\pm\sqrt{2}}{2}$ ✓a. (3)

5. $\frac{2}{(x-1)(x+1)}-\frac{1}{x(x-1)}=\frac{2}{x^2}$ ✓ ✓

LCD $x^2(x-1)(x+1)$

$x\neq 0, 1$ or -1 ✓

$2x^2-(x+1)=2(x-1)(x+1)$ ✓

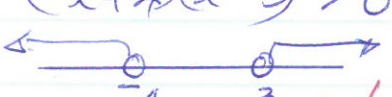
$2x^2-x-1=2x^2-2$ ✓

$1=x$ ✓

no solutions ✓ (6)

(b) $x^2+x-12>0$

$(x+4)(x-3)>0$



$x\in(-\infty;-4)$ ✓ or $x\in(3;\infty)$ ✓ (3)

(c) $2x^2+5x-4=0$

$x^2+\frac{5}{2}x = 2$ ✓m

$x^2+\frac{5}{2}x+\frac{25}{16} = 2+\frac{25}{16}$ ✓m.

$(x+\frac{5}{4})^2 = \frac{47}{16}$ ✓m ea.

$x+\frac{5}{4} = \pm\frac{\sqrt{47}}{4}$

$x = \frac{-5\pm\sqrt{47}}{4}$ ✓a. (4)

2. (a) $x^2+y^2-2=0$ (1)

$x+2y-1=0$ (2)

from (2) $x=1-2y$ (3) } ✓m.

Sub this into (1)

$(1-2y)^2+y^2-2=0$

$1-4y+4y^2+y^2-2=0$

$5y^2-4y-1=0$ ✓a

$(5y+1)(y-1)=0$ ✓ca.

$y=-\frac{1}{5}$ ✓ or $y=1$ ✓ca

Sub into (3)

$x=1-2(-\frac{1}{5})$ or $x=1-2(1)$

$=1+\frac{2}{5}$ ✓ $x=-1$

$=\frac{7}{5}$. $(-1;1)$ ✓ (6)

(b) $f(x)=x^2-5x+c$

roots 3 and 2 $f(x)=(x-3)(x-2)$

$=x^2-5x+6$

$c=6$. ✓m. (1)

(a) 1. $x^2 + x = 3x - k - 2$

$x^2 - 2x + k + 2 = 0$ ✓

$b^2 - 4ac = 4 - 4(k+2)$ ✓✓

$= 4 - 4k - 8$

$= -4k - 4$ (2)

2. Sub $k = -10$ into Δ.

$\Delta = 40 - 4$ ✓

$= 36$.

Roots real, rational and unequal. ✓✓ (2)

3. Real roots $b^2 - 4ac \geq 0$. ✓

$-4k - 4 \geq 0$ ✓

$-4k \geq 4$

$k \leq -1$. ✓

Max value for $k = -1$. ✓ (2)

3. (or 1. $\dfrac{4^{n+1} \cdot 8^{2n}}{16^{2n-1}} = \dfrac{2^{2n+2} \cdot 2^{6n}}{2^{8n-4}}$ ✓ ✓ m

$= 2^{2n+2+6n-8n+4}$ ✓ m/a

$= 2^6$ ✓ ca. $= 64$ (3)

2. $\dfrac{6 \cdot 2^{2n+3} - 8 \cdot 2^{2n-1}}{2 \cdot 2^{2n+2}}$

$= \dfrac{2^{2n}(6 \cdot 2^3 - 8 \cdot 2^{-1})}{2^{2n+3}}$ ✓ ✓ ✓

$= 2^{-3}(48 - 4)$ ✓

$= \dfrac{44}{8}$ ✓ $= 5{,}5$ (4)

3. $\dfrac{\sqrt{300} - \sqrt{75}}{\sqrt{48}} = \dfrac{10\sqrt{3} - 5\sqrt{3}}{4\sqrt{3}} = \dfrac{5\sqrt{3}}{4\sqrt{3}}$ ✓

$= \dfrac{5}{4}$ ✓ (3)

(b) 1. $3^{2x-1} = 27^{2x-1}$

$3^{2x-1} = 3^{6x-3}$ ✓m

$2x-1 = 6x-3$ ✓m

$2 = 4x$

$x = \frac{1}{2}$ ✓a. (2)

2. $2^{x+1} + 2^x - 2^{x-1} = 20.$

$2^x(2^1 + 1 - 2^{-1}) = 20$

$2^x(3 - \frac{1}{2}) = 20$

$2^x = 20 \div \frac{5}{2}$

$= \cancel{20}^{4} \times \frac{2}{\cancel{5}}$ ✓

$2^x = 2^3$

$x = 3.$ ✓ (4)

4. (a) 1. function ✓ m-1 ✓

2. not a function m-m ✓

3. not a function 1-m ✓

4. function ✓ 1-1 ✓ (4)

(b) $f(x) = x^2 - 3x + 1 \quad g(x) = -2x^2 - 1.$

1) $f(1) + g(1)$

$= 1^2 - 3 + 1$ ✓ $+ \quad 2(1)^2 - 1$ ✓

$= 1 - 3 + 1 - 2 - 1$ ✓.

$= -4$ ✓ (2)

2. $f(x) = 11$

$x^2 - 3x + 1 = 11$ ✓m

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$ ✓a

$x = 5$ or -2 ✓ca. (3)

5. 1. $y = x^2$ given

2. $y = x^2 + 3x + 2$. ✓
3. $y = -2(x-1)^2 - 1$. ✓
4. $y = (x-2)^2$. ✓
5. $y = \frac{1}{x-1} - 1$. ✓
6. $y = 2y = x + 2$. ✓
7. $y = -\frac{1}{x} + 1$. ✓
8. $2y + x - 2 = 0$ ✓ (7)

6. (a) $f(x) = \frac{a}{x+p} + q$. x intercept 1. $x = -1$

$f(x) = \frac{a}{x+1} - 1$ ✓ ✓

Sub $(-1; 0)$ $0 = \frac{a}{2} - 1$ ✓

$1 = \frac{a}{2}$

$2 = a$ ✓

$p = 1$ ✓ $q = -1$ ✓ $a = 2$. (5)

(b) $y = (x+1) - 1$ ✓

$y = x$. ✓ (2)

(c) Domain $x \in \mathbb{R}$ ✓ but $x \neq -1$ ✓ (2)

7. new page.

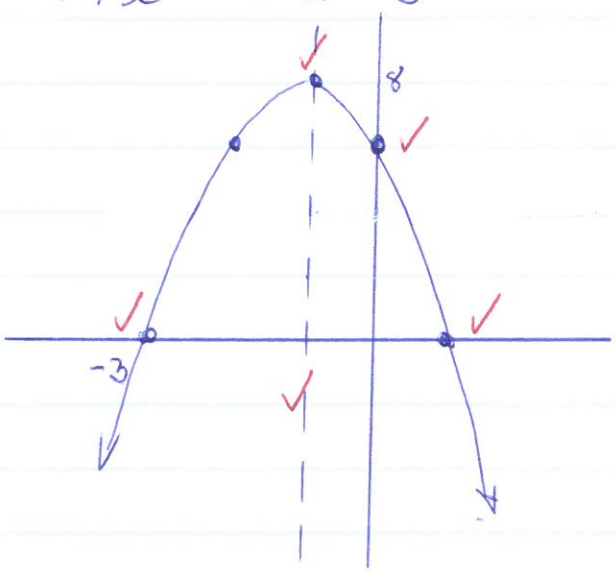
7. $f(x) = -2(x+1)^2 + 8$

(a)
- a − ∩ max y value of 8
- TP $(-1; 8)$
  Adj S $x = -1$
- Y int $(0; 6)$ $y = -2(0+1)^2 + 8$
  $= -2 + 8$
  $= 6$
- X int $y = 0$ $0 = -2(x+1)^2 + 8$
  $4 = (x+1)^2$

either $\pm 2 = x+1$ or $x^2 + 2x + 1 - 4 = 0$

$\therefore x = 1$ or -3 $(x-1)(x+3) = 0$

$x = 1$ or -3.



Shape ✓ (6)

(b) 1. max value of 8.
2. increasing for $x \in (-\infty; -1)$.

7 (a) $p(x) = ax^2 + bx + c$.

x-int. -1 or 3

$y = a(x+1)(x-3)$ ✓

Sub. $(0;-1)$ $\quad -1 = a(1)(-3)$

$\frac{1}{3} = a$ ✓

$y = \frac{1}{3}(x^2 - 2x - 3)$

$y = \frac{1}{3}x^2 - \frac{2}{3}x - 1$ ✓

(3)

(b) $y = mx + c$ ✓

$y = -\frac{1}{3}x + 1$ ✓

(2)

(c) $y = \frac{1}{3}x^2 - \frac{2}{3}x - 1$

$y = -\frac{1}{3}x + 1$

$\frac{1}{3}x^2 - \frac{2}{3}x - 1 = -\frac{1}{3}x + 1$ ✓

$\frac{1}{3}x^2 - \frac{1}{3}x - 2 = 0$

$x^2 - x - 6 = 0$ ✓

$(x-3)(x+2) = 0$ ✓

$x = 3$ or -2

A x coordinate $x = -2$ ✓

(4)

(d) Axis of Symmetry. $x = 1$ ✓

Sub into original

$y = \frac{1}{3}(1)^2 - \frac{2}{3}(1) - 1$

$= \frac{1}{3} - \frac{2}{3} - 1$

$= -\frac{4}{3}$ ✓

Range: $y \in [-\frac{4}{3}; \infty)$ ✓

(2)