ASHTON
INTERNATIONAL COLLEGE

BALLITO

Mathematics Paper 1
FORM 4
12th November 2015

TIME: 3 hours	TOTAL: 150 marks

Examiner: Mrs A Gunning	Moderated: Mr R Steenhuisen

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY BEFORE ANSWERING THE QUESTIONS.

- This question paper consists of 11 pages, which includes an information sheet. Please check that your question paper is complete.

- Read and answer all questions carefully.

- It is in your own interest to write legibly and to present your work neatly.

- Number your answers exactly as the questions are numbered.

- All necessary working which you have used in determining your answers **must** be clearly shown.

- Approved non-programmable calculators may be used except where otherwise stated. Where necessary give answers correct to 2 decimal places unless otherwise stated.

- Diagrams have not necessarily been drawn to scale.

SECTION A
QUESTION 1

Solve for x in each of the following. Where relevant, leave your answers in the simplest surd form.

(a) $(x - 3)(x + 4) = 18$ (3)

(b) $\dfrac{30}{x-2} - \dfrac{1}{2} = \dfrac{30}{x}$, stating any relevant restrictions to the values of x.   (5)

(c) $2^{x-1}.8^{x+1} = 16^{-x-1}$ (4)

(d) $x = 2 - \sqrt{2x - 5}$ (5)

[17]

QUESTION 2

(a) For which values of b will the expression $\dfrac{b^2 - 5b + 6}{b + 2}$ be:

  (1) Undefined   (1)

  (2) Equal to zero   (2)

(b) Determine the value of k and the other root of the equation $kx^2 - 7x + 4 = 0$, given that one of the roots of the equation is 1.   (4)

(c) Solve the following equations simultaneously

    $x - 2y = 1$ and $x^2 + y^2 = 29$ (6)

(d) Given the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.   (4)

[17]

QUESTION 3

(a) Given that $(2 + \sqrt{7})(4 - \sqrt{7}) = a + b\sqrt{7}$, where a and b are integers, find the value of a and the value of b.   (2)

(b) Simplify each of the following without the use of a calculator. Remember you must show all relevant working details.

  (1) $\dfrac{\sqrt{50} - \sqrt{8}}{\sqrt{72}}$ (3)

  (2) $\left(\dfrac{2x^2}{16x^{-1}}\right)^{\frac{1}{3}}$ (3)

Question 3(b) continued

(3) $\sqrt{\dfrac{2^{m+5}+2^{m+2}}{2^{m-2}}}$ (4)

[12]

QUESTION 4

(a) How much money would you have to invest if you wanted to have R500 000 after 5 years? The interest rate is 15% p.a. calculated semi-annually. (3)

(b) Calculate the effective annual interest rate for an interest rate of 12,5% p.a. compounded monthly. (3)

[6]

QUESTION 5

In a survey, 100 students were asked if they like basketball (B), football (F) and swimming (S). The Venn diagram shows the results.
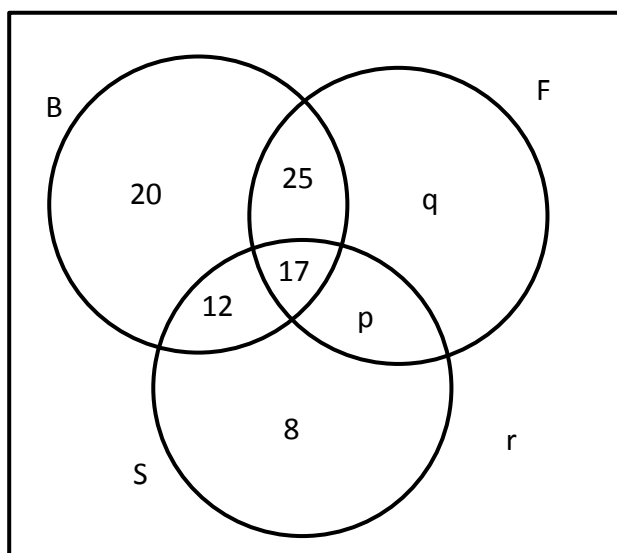

42 Students like swimming.

40 students like exactly 1 sport.

(a) Find the values of *p, q* and *r*. (3)

(b) One student is chosen at random from the 100 students. Find the probability that the student:

(1) Only likes swimming (1)

(2) Likes basketball but not swimming. (1)

(c) A student is chosen at random from those who like basketball. Find the probability that he likes exactly one other sport. (2)

[7]

QUESTION 6
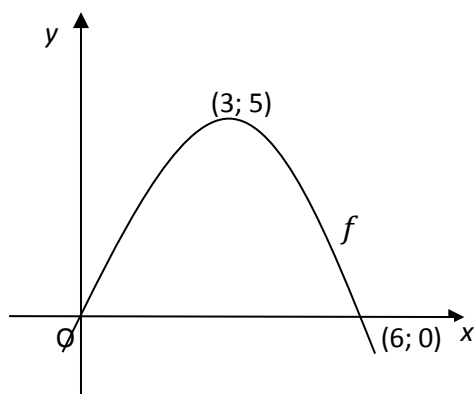
The above diagram shows a sketch of the curve with equation $y = f(x)$. The curve passes

through the origin O and through the point $(6; 0)$. The maximum point on the curve is $(3; 5)$.

Using the information given in the sketch above, draw each of the following on a separate

set of axes on the graph paper provided in the answer booklet.

(a) $y = 3f(x)$ (2)

(b) $y = f(x + 2)$ (2)

(On each diagram, show clearly the coordinates of the maximum point and of each point at

which the curve crosses the x-axis.) **[4]**

QUESTION 7

Given that $f(x) = x^2 - 6x + 18$ for $x \geq 0$

(a) Express $f(x)$ in the form $f(x) = (x - p)^2 + q$, where p and q are integers. (3)

(b) The curve with equation $y = f(x)$ $x \geq 0$, meets the y-axis at R and has a minimum

point at T. Sketch the graph of f, showing the coordinates of R and T. (4)

(c) Draw the line $y = 15$ on the same set of axes, to meet f at the points M and N.

Calculate, algebraically the x-coordinates of M and N, leaving your answers in the

simplest surd form. (3)

[10]

QUESTION 8

(a) Write down the next 2 terms and determine an expression for the nth term of the

sequence   5; 12; 23; 38;   …; ….   (6)

(b) A quadratic sequence has a second term equal to 1, a third term equal to -6 and the 4th

term equal to -14.

    (1) Determine the 2nd difference for this sequence.   (2)

   (2) Hence or otherwise calculate the first term of the pattern.   (2)

[10]

SECTION B

QUESTION 9

(a) Determine the domain of $g(x) = \frac{2}{x+1} + 2$ (2)

(b) Write down the range of $h(x) = 2^{x+1} - 3$ (2)

(c) Give the equation of the positive axis of symmetry for

$$j(x) = \frac{1}{x-1} - 2$$ (2)

(d) The point of intersection of the 2 asymptotes of $f(x) = \frac{1}{x-p} + q \ is \ (2; 1).$ State the

values of p and q.   (2)

(e) Determine the x-intercept and the y-intercept of $y + 5^{x+2} = 5$ (3)

(f) Find the equation for each of the functions sketched below. $f(x) = 2^x + q$ and $g(x) = mx + c$. The graphs intersect at the point $(0; -\frac{1}{2})$. g crosses the x-axis at the point $(-6; 0)$. (3)
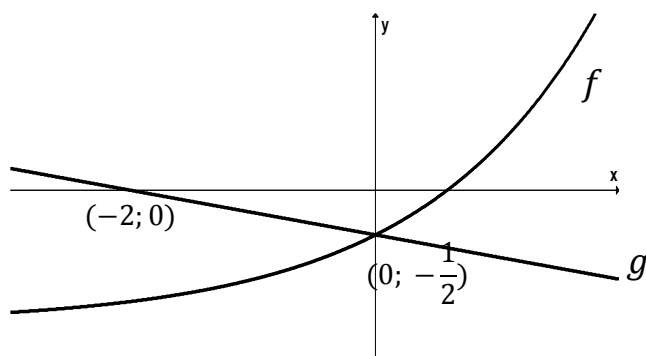
(g)
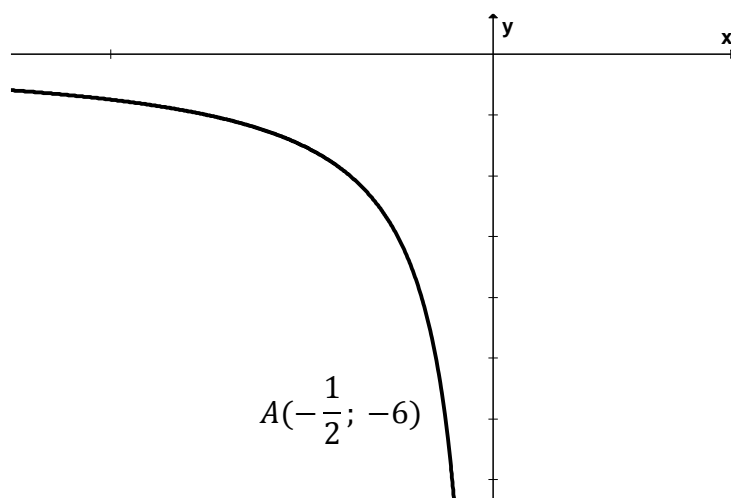
Sketched above is the graph of the hyperbola $h(x) = \frac{k}{x}$ for $x < 0$. The graph passes through the point $A(-\frac{1}{2}; -6)$.

(1) Show that $k = 3$. (1)

(2) Write down the equation of the new function formed if $h(x)$ is:

    a. Shifted 3 units vertically upwards. (1)

    b. Reflected about the x axis. (1)

    c. Shifted so that the asymptotes are $x = 1$ and $y = -2$ (2)

[19]

QUESTION 10

(a) Solve for *x*:

$$x^2 + x - 3 + \frac{1}{x^2+x-3} = -2 \tag{6}$$

(b) A person builds a rectangular storeroom. If the diagonal of the room is $\sqrt{1312}\ m$ and

the perimeter is $80\ m$, determine the dimensions of the room. (5)

(c) Find the set of values of x for which:

    (1) $3(2x + 1) > 5 - 2x$ (2)

    (2) $2x^2 - 7x + 3 \geq 0$ (4)

    (3) Both $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 \geq 0$ (2)

[19]

QUESTION 11

(a) Mandy starts saving by depositing R5 000 into an account in January 2010. She deposits

a further R6 000 into the account 2 years later. 3 years after that, she has a cash-flow

problem and withdraws R3 000. The interest rate for the first 3 years is 8% p.a.

compounded monthly, but changes to 9% p.a. compounded quarterly for the rest of the

investment period. Make use of a timeline to assist you in calculating how much money

will she have in the account in January 2017? (6)

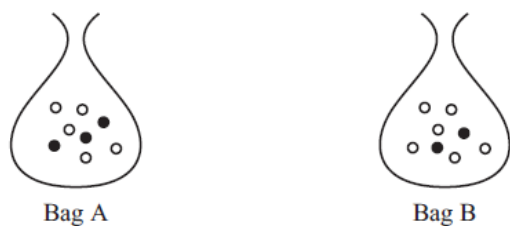(b) The graph below shows the depreciating value of a car over a period of time.

(c)
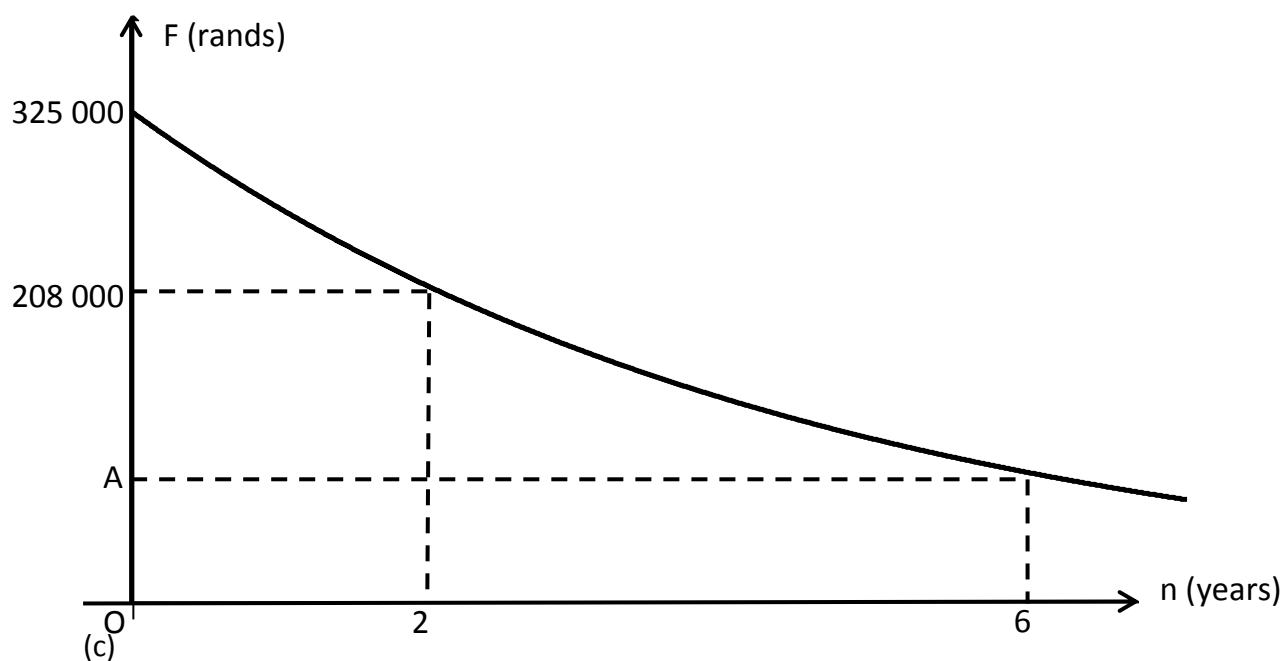
   (1) What is the cost of the car when new?   (1)

   (2) What type of depreciation is illustrated? (1)

   (3) Use the information on the graph to calculate the rate of depreciation. (3)

   (4) Calculate A, the value of the car after 6 years. (2)

[13]

QUESTION 12

(a) William must choose a ball from Bag A or from Bag B.
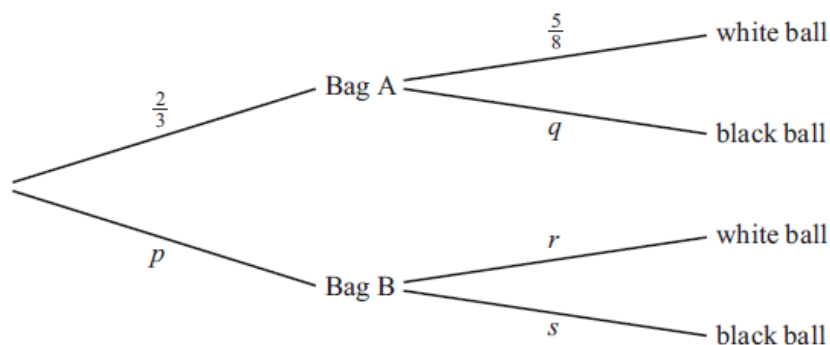

Bag A   Bag B

The probability that he chooses Bag A is $\frac{2}{3}$.

Bag A contains 5 white and 3 black balls.

Bag B contains 6 white and 2 black balls.

The tree diagram below shows some of this information.

(1) Find the values of p, q, r and s. (3)

(2) Find the probability that William chooses Bag A and then a white ball. (2)

(3) Find the probability that William chooses a white ball. (2)

(b) Another bag contains 7 green balls and 3 yellow balls. Sam takes three balls out of the bag, without replacing them.

(1) Find the probability that all 3 balls he chooses are yellow. (2)

(2) Find the probability that at least one of the balls he chooses is green. (1)

[10]

QUESTION 13

You are given the graphs of $k(x) = -x^2 + 3x + 10$ with turning point at D. The graph of

the straight line h passes through the points B and C.
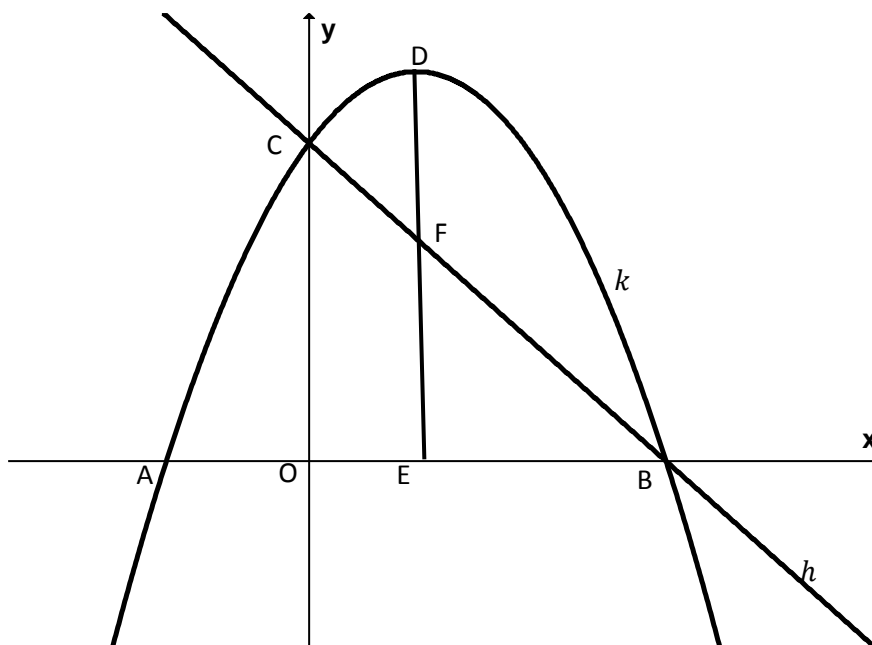
(a)   Write down the equation of the line DE   (2)

(b)   Give the values of x for which $k(x) < 0$ (3)

(c)   Give the x values for which $k(x) \geq h(x)$ (1)

[6]

Form 4

Mathematics Paper 1

11 November 2015

This question is to be done in place of **QUESTION 8** on page 5**.**

QUESTION 8

(a) Consider $2n$

     a. Explain why 2n is an even number.   (1)

     b. Explain why 2n-1 is an odd number.   (1)

     c. What is the next odd number after 2n-1?   (1)

     d. Prove that the **difference** of the **squares** of the 2 consecutive odd numbers

        (from part b and c), is a multiple of 8.   (3)

(b) Consider $(x + y)^2$

    a. Simplify $(x + y)^2$ (1)

    b. Using your answer in part 1 and the fact that $x + y = 6$ and $xy = 7$, find the value of $x^2 + y^2$ (3)

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad\qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad\qquad A = P(1 - i)^n$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad\qquad y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad m = \tan\theta$$

$$In \; \triangle ABC: \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \, \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\bar{x} = \frac{\sum f x}{n} \qquad\qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad\qquad P(A \; or \; B) = P(A) + P(B) - P(A \; and \; B)$$